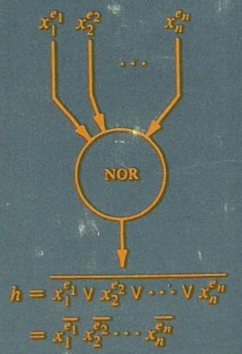
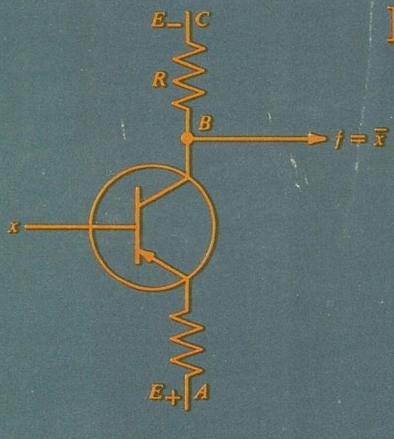
Franz E. Hohn UNIVERSITY OF ILLINOIS



Applied Boolean Algebra

An Elementary Introduction



SECOND EDITION

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An Elementary Introduction

FRANZ E. HOHN
Professor of Mathematics
University of Illinois

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The Algebra of Combinational Relay Circuits

The Mathematical Model

1.1 Bistable Devices

In electronic digital computers, telephone switching systems, control systems for automatic factories, and other systems involving the communication or processing of data, we find many examples of electric circuits that employ what are known as **two-state** or **bistable devices.** The simplest example of such a device is a **switch** or **contact** that may be in the **open state** or in the **closed state.** When a contact is operated with the aid of an electromagnet, the combination is called a **relay.** A switch or relay is called a **bilateral circuit element** since it permits the passage of current in either direction when the contact is closed. Devices permitting the passage of current in only one direction are called **unilateral.**

There are various other two-state devices in use or in the process of development. These include rectifying diodes, magnetic cores, transistors, various types of electron tubes, cryotrons, and a variety of others. Magnetic drums and magnetic tapes may be regarded as assemblages of two-state devices. The physical nature of the two stable states of a device varies from one device to another and may take such forms as conducting vs. nonconducting, closed vs. open, charged vs. discharged, positively magnetized vs. negatively magnetized, high potential vs. low potential, and other states.

The methods and results of Boolean algebra and related subjects, such as logic and set theory, have been found useful in discussing circuits that employ two-state devices. In this chapter we use contact networks to illustrate how this is done. The mathematical model is particularly simple in this case and, historically, this is the first application of Boolean algebra to digital circuitry (Shannon: [1], [2]).

Some commonly used symbols for contacts are shown in Figure 1.1.1. In this book we often use the first type of symbol, but frequently without representation of the electromagnet that operates the contact. Each of

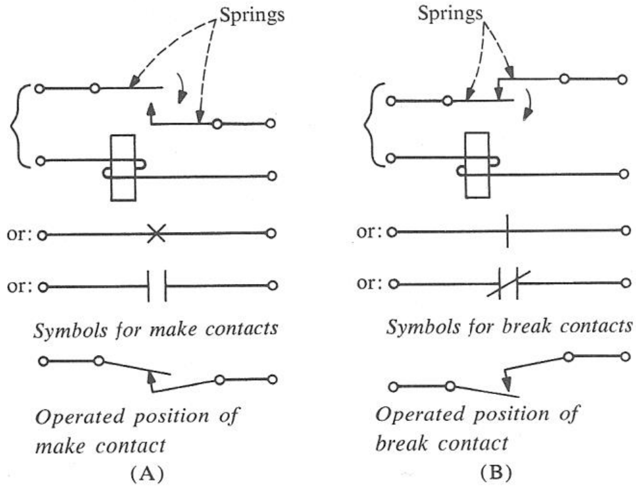


Figure 1.1.1

the contacts illustrated here has two flexible blades called **springs.** One of these springs is forced by the attraction of the electromagnet to move toward or away from the other when current is passed through the coil around the iron core of the magnet. A normally open contact is called a **make** or **front contact** whereas a normally closed one is called a **break** or **back contact.** The wires that conduct (lead) the current through the network are called **leads.** Symbols for other devices are given later as needed.

1.2 Circuit Variables and Their Complements

We introduce two mathematical symbols as the first step in constructing the announced model. With an open contact (or open path) in a circuit we associate the symbol "0" and with a closed contact (or closed path) we associate the symbol "1". Although we call these symbols **zero** and **one,** respectively, they are not to be regarded as the zero and one of ordinary arithmetic. From a mathematical point of view, "0" and "1" are to be regarded as *undefined terms.* The definitions to follow and the postulates of Section 1.6 give 0 and 1 their mathematical meaning. All we have stated here is the particular *physical interpretation* which it is convenient to give to 0 and 1 in this application.

When the condition of a contact is variable in a problem, we represent it by a literal symbol such as x, y, a, b, and so on. Such a symbol, called a **circuit variable,** assumes the value 0 when the contact is open, the value 1 when it is closed.

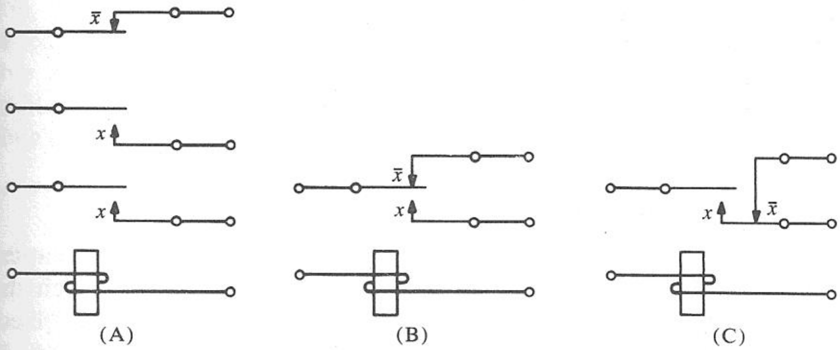
With each symbol x, we associate a symbol $\bar{x}$, called the **complement** of x, which assumes the value 1 when x assumes the value 0, the value 0 when x assumes the value 1:

x	$\bar{x}$
0	1
1	0

Note that we have in fact defined an **operation of complementation** in terms of the symbols 0 and 1. This is the first step in the process of giving these symbols their mathematical meaning. The definition of the complement is intended to imply that

$$\bar{0} = 1, \quad \bar{1} = 0. \tag{1.2.1}$$

The complement $\bar{x}$ of x is employed as the circuit variable associated with a contact which is open when the x-contact is closed and is closed when the x-contact is open. Thus, if x denotes a normally open contact of a certain relay, $\bar{x}$ denotes a normally closed contact of that same relay, as is shown in all three parts of Figure 1.2.1. *Throughout this chapter, an uncomplemented variable refers to a normally open (make) contact and a complemented variable refers to a normally closed (break) contact.*



Contacts operated by the same coil

Break-before-make or transfer contact

Make-before-break contact or continuity transfer

Figure 1.2.1