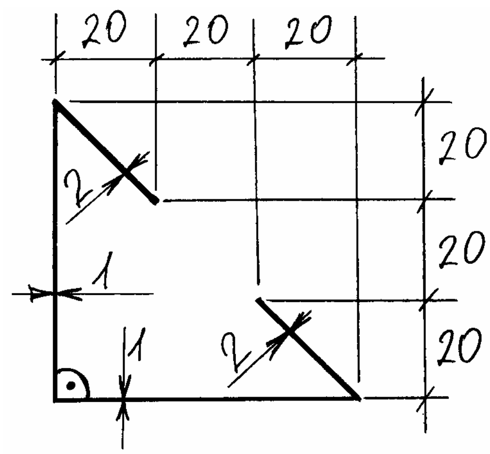
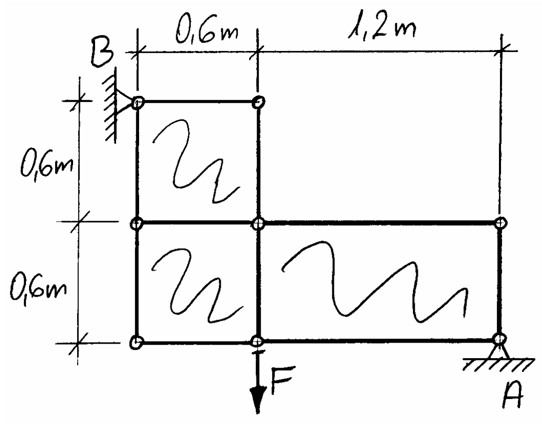
1. Feladat *(25 pont)*:



a) Határozza meg a vázolt szelvény M nyírási középpontjának helyét!
b) Határozza meg a keresztmetszet $S_{\alpha\omega}$ függvényét a jellemző értékek feltüntetésével!
c) Határozza meg a keresztmetszet I_t torziós másodrendű nyimatékát!

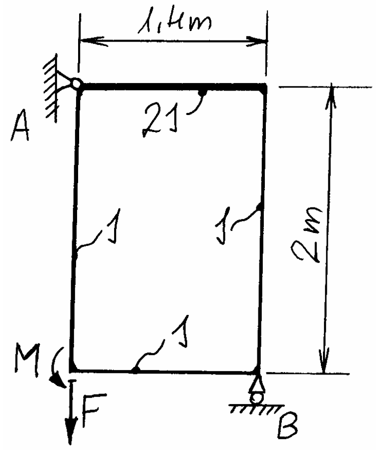
2. Feladat *(25 pont)*: A vázolt lemezzel merevített szerkezet minden rúdja azonos anyagú (E) és azonos keresztmetszetű (A_K), a lemez vastagsága „v". A tartót a koncentrált F erő terheli.



a) Határozza meg a rudak normál-igénybevételeit és a lemezek nyírófolyam-igénybevételeit!

Adatok: $A_K = 400\,\text{mm}^2$; $E = 2,6 \cdot G = 200\,\text{GPa}$; $v = 0,5\,\text{mm}$; $F = 5\,\text{kN}$

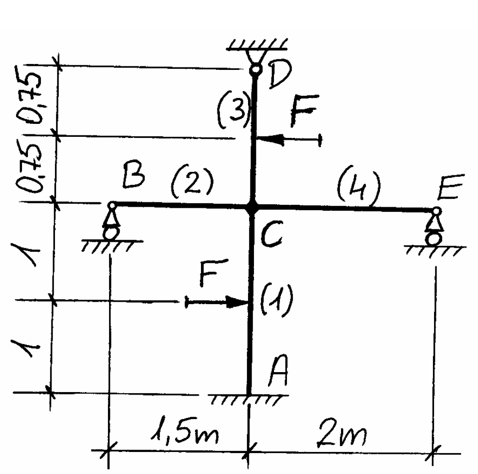
3. Feladat *(25 pont)*: A vázolt zárt keret minden rúdja azonos anyagú (E). A tartót a koncentrált F erő és M nyomaték terheli.



a) σ-ponti módszerrel határozza meg a keret hajlító nyomatéki ábráját a jellemző értékek feltüntetésével!

Adatok: $F = 2000\,\text{N}$; $M = 1200\,\text{Nm}$; $E = \text{áll.}$

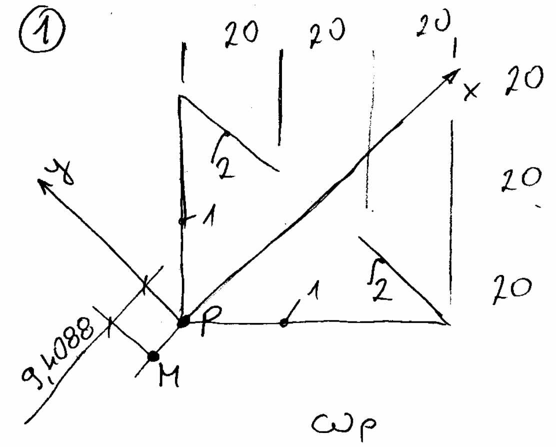
4. Feladat *(25 pont)*: A vázolt törttengelyű tartó minden rúdja azonos anyagú (E=áll.), húzó-nyomó merevsége végtelen, a rudak másodrendű nyomatékai adottak és a két darab F koncentrált erő terheli.



a) Mozgásmódszerrel határozza meg a tartó hajlító igénybevételi ábráját!

Adatok: $I_1 = I_3 = I$; $I_2 = I_4 = 0,8 \cdot I$; $F = 4800\,\text{N}$; $AE = \infty$

1



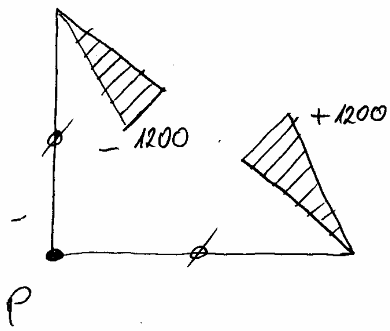
$$\eta_M = 0$$

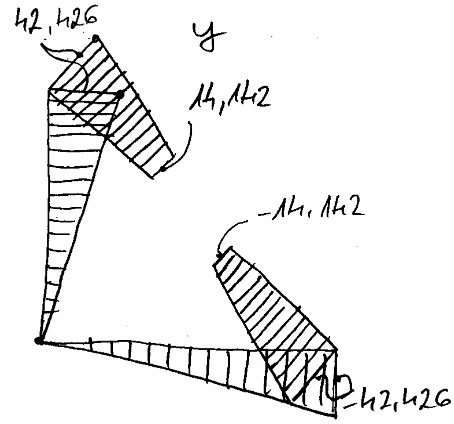
$$\xi_M = \frac{1}{I_x}\int \omega_P \cdot y \cdot v \, du$$

$$I_x = 2\cdot\left[1\cdot 60\cdot\left[\frac{\left(\frac{60}{\sqrt{2}}\right)^2}{12} + \left(\frac{30}{\sqrt{2}}\right)^2\right] + 2\cdot 20\sqrt{2}\left[\frac{(20\sqrt{2})^2}{12} + (20\sqrt{2})^2\right]\right] =$$

$$= 2\cdot[36000 + 49026,07] = 170052,14 \text{ mm}^4$$

ω_P

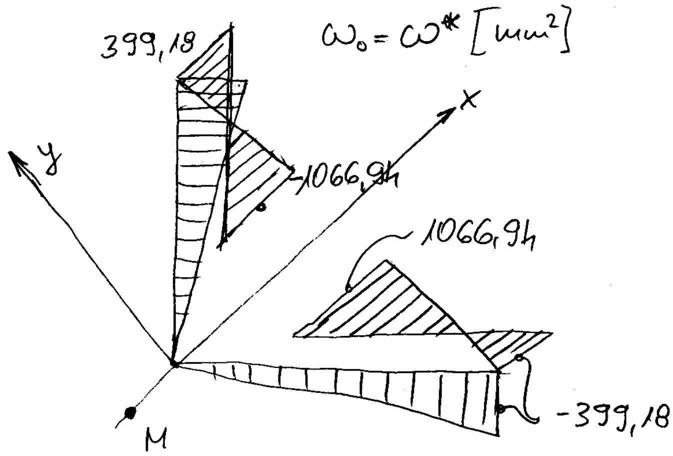




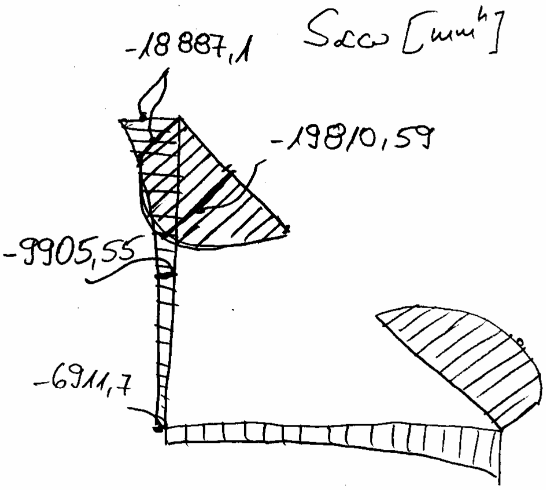
$$\int \omega_P \cdot y \cdot v \, du = 2\cdot\left[2\cdot\frac{20\sqrt{2}}{6}\left(0 + 4\cdot -600\cdot 28,284 - 1200\cdot 14,142\right)\right] = -1\,599\,987,725 \text{ mm}^5$$

$$\xi_M = -9,4088 \text{ mm}$$

$\omega_0 = \omega^* \ [\text{mm}^2]$

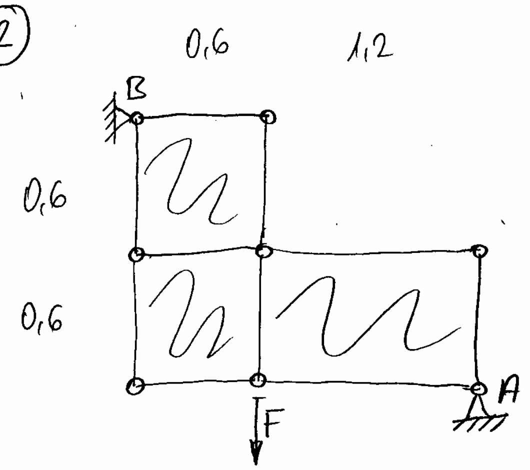


$S_{x\omega}\ [\text{mm}^4]$

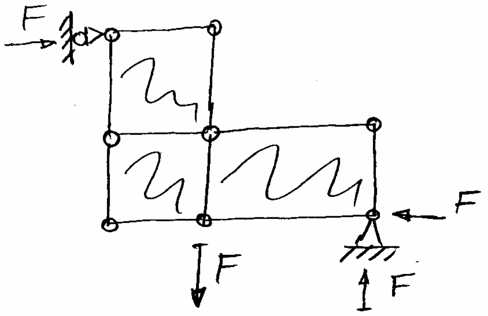


$$I_t = \frac{1}{3}\cdot\left[2^3\cdot 20\cdot\sqrt{2} + 1^3\cdot 60\right]\cdot 2 = 190,85 \text{ mm}^4$$

(2)



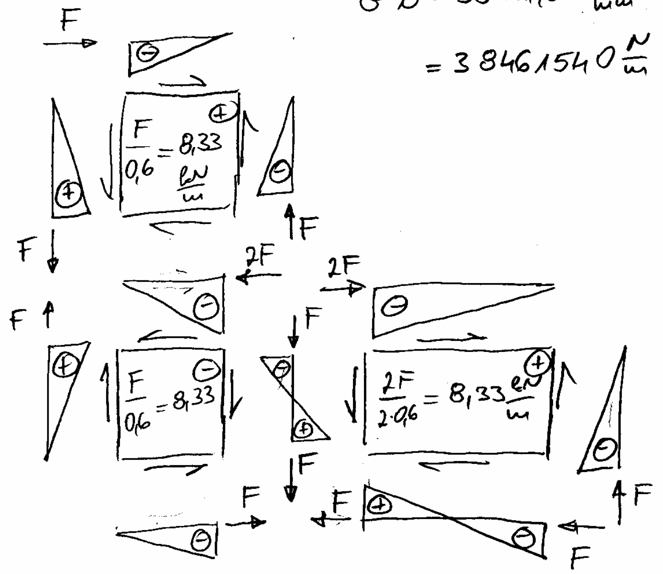
1-szeresen határozatlan!



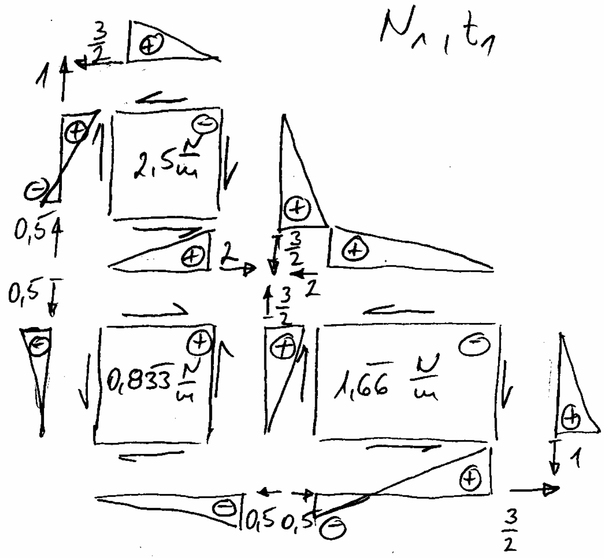
$AE = 8 \cdot 10^7\ N$

$G \cdot \nu = 38461{,}54 \frac{N}{mm} = 3846154{,}0 \frac{N}{m}$

$N_0,\ t_0$



$N_1,\ t_1$



$$\delta_{10} = \int \frac{N_0 \cdot N_1}{A \cdot E} du + \int \frac{t_0 \cdot t_1}{G \cdot \nu} dA = \frac{1}{AE}\left[\frac{F \cdot 0{,}6}{2} \cdot 0 + \frac{F \cdot 0{,}6}{2} \cdot -\frac{2}{3} \cdot 0{,}5 - \frac{F \cdot 0{,}6}{2} \cdot \frac{2}{3} \cdot \frac{3}{2} - \frac{2F \cdot 0{,}6}{2} \cdot \frac{2}{3} \cdot 2 + \frac{F \cdot 0{,}6}{2} \cdot \frac{2}{3} \cdot 0{,}5 - \right.$$

$$- \frac{F \cdot 0{,}6}{2} \cdot \frac{2}{3} \cdot \frac{3}{2} - \frac{3}{2} \cdot \frac{0{,}6}{2} \cdot \frac{F}{3} - \frac{2F \cdot 1{,}2}{2} \cdot \frac{2}{3} \cdot 2 + \frac{1{,}2}{6}\left(-\frac{F}{2} + 4 \cdot 0 - \frac{3F}{2}\right) - \left.\frac{F \cdot 0{,}6}{2} \cdot \frac{2}{3} \cdot 1\right] + \frac{1}{G \cdot \nu}\left[0{,}6^2 \cdot -\frac{F}{0{,}6} \cdot 2{,}5 - \right.$$

$$\left. - 0{,}6^2 \cdot \frac{F}{0{,}6} \cdot 0{,}8\overline{33} - 1{,}2 \cdot 0{,}6 \cdot \frac{F}{0{,}6} \cdot 1{,}\overline{66}\right] = -\frac{3{,}75 F}{AE} - \frac{4F}{G \cdot \nu} = -7{,}544 \cdot 10^{-4}\ m$$

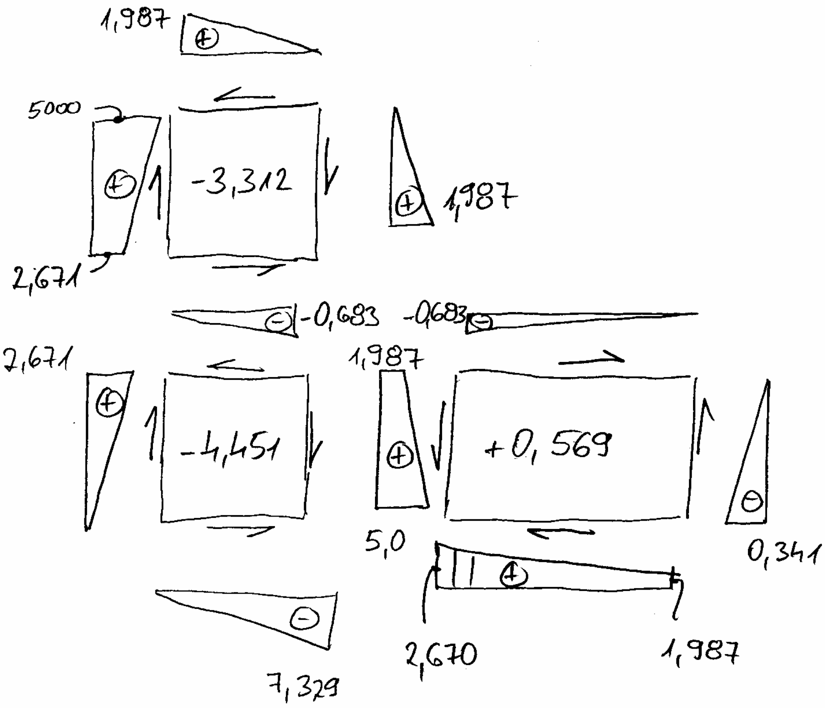
$$\delta_{11} = \frac{1}{AE}\left[\frac{0{,}6}{6}\left(1^2 + 4 \cdot 0{,}25^2 + 0{,}5^2\right) + \frac{0{,}5 \cdot 0{,}6}{2} \cdot \frac{2}{3} \cdot 0{,}5 + \frac{3 \cdot 0{,}6}{2 \cdot 2} \cdot \frac{2}{3} \cdot \frac{3}{2} + \frac{2 \cdot 0{,}6}{2} \cdot \frac{2}{3} \cdot 2 + \frac{0{,}5 \cdot 0{,}6}{2} \cdot \frac{2}{3} \cdot \frac{1}{2} + \frac{3 \cdot 0{,}6}{2 \cdot 2} \cdot \frac{2}{3} \cdot \frac{3}{2} + \right.$$

$$\left. + \frac{3 \cdot 0{,}6}{2 \cdot 2} \cdot \frac{2}{3} \cdot \frac{3}{2} + \frac{2 \cdot 1{,}2}{2} \cdot \frac{2}{3} \cdot 2 + \frac{1{,}2}{6}\left(0{,}5^2 + 4 \cdot 0{,}5^2 + \frac{9}{4}\right) + \frac{1 \cdot 0{,}6}{2} \cdot \frac{2}{3} \cdot 1\right] + \frac{1}{G \cdot \nu}\left[0{,}6^2 \cdot 2{,}5^2 + \right.$$

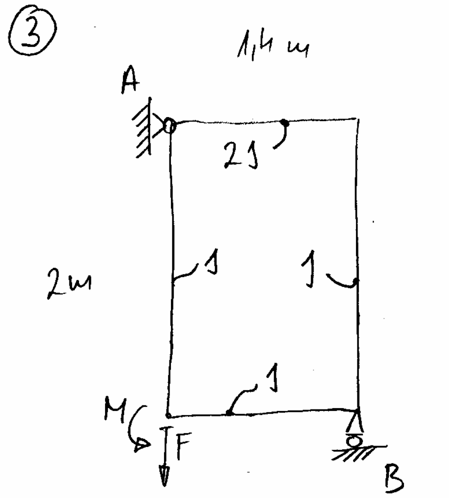
$$\left. + 0{,}833^2 \cdot 0{,}6^2 + 1{,}\overline{66}^2 \cdot 1{,}2 \cdot 0{,}6\right] = \frac{3{,}7}{AE} + \frac{4{,}45}{G \cdot \nu} = 1{,}6195 \cdot 10^{-7}$$

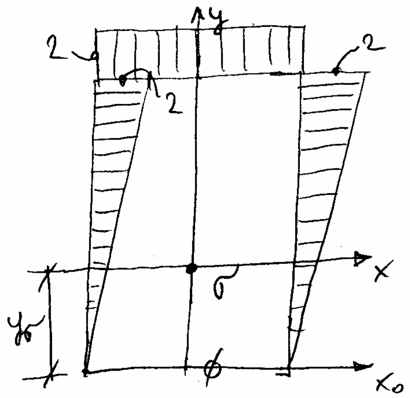
$$\delta_{10} + X_1 \cdot \delta_{11} = 0 \qquad X_1 = -\frac{\delta_{10}}{\delta_{11}} = \frac{+7{,}544 \cdot 10^{-4}}{1{,}6195 \cdot 10^{-7}} = 4658{,}23\ N$$

2) $N = N_0 + X_1 \cdot N_1 \,[kN]$ $\quad t = t_0 + t_1 \cdot X_1 \left[\frac{kN}{m}\right]$



3

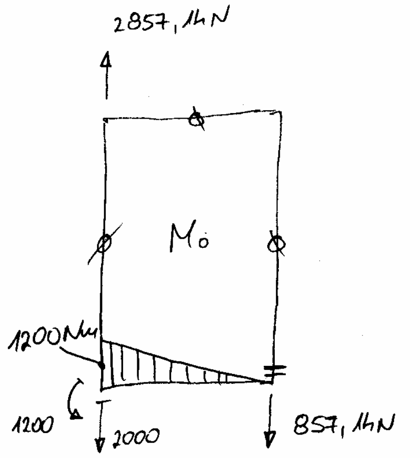


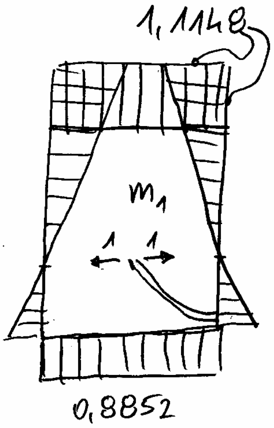


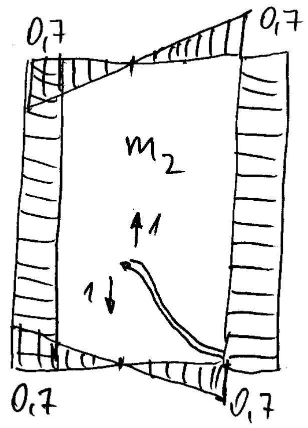
$$S^*_{x0} = \int \frac{y}{IE}\,ds = \frac{1}{IE}\left[\frac{2\cdot 2}{2} + 0 + \frac{2\cdot 2}{2}\right] + \frac{1}{2IE}\left[2\cdot 1{,}4\right] = \frac{5{,}4}{IE}$$

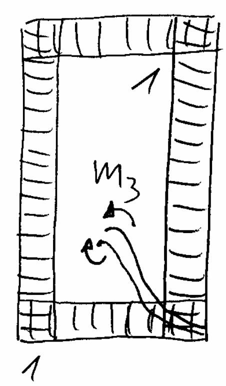
$$y_\sigma = \frac{S^*_{x0}}{L_S} = \frac{5{,}4}{6{,}1} = 0{,}8852 \text{ m}$$

$$L_S = \int \frac{ds}{IE} = \frac{1}{IE}\left[2 + 1{,}4 + 2\right] + \frac{1}{2IE}\left[1{,}4\right] = \frac{6{,}1}{IE}$$









$$\delta_{10} = \int \frac{M_0\cdot m_1}{IE}\,ds = \frac{1}{IE}\left[-\frac{1200\cdot 1{,}4}{2}\cdot 0{,}8852\right] = -\frac{743{,}568}{IE}$$

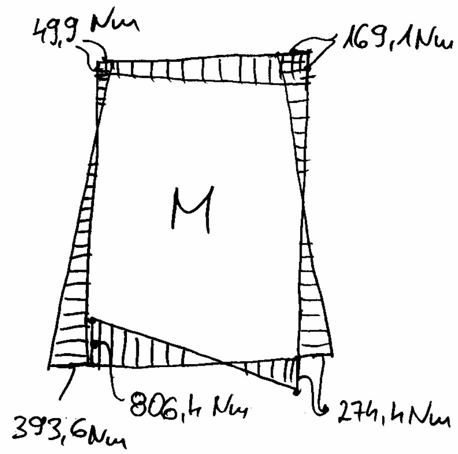
$$\delta_{20} = \int \frac{M_0\cdot m_2}{IE}\,ds = \frac{1}{IE}\left[\frac{1{,}4}{6}\left(1200\cdot 0{,}7 + 4\cdot 0 + 0\right)\right] = \frac{196}{IE}$$

$$\delta_{30} = \int \frac{M_0\cdot m_3}{IE}\,ds = \frac{1}{IE}\left[\frac{1200\cdot 1{,}4}{2}\cdot 1\right] = \frac{840}{IE}$$

$$\delta_{11} = \int \frac{m_1\cdot m_1}{IE}\,ds = \frac{1}{IE}\Big[2\cdot\frac{2}{6}\left(1{,}1148^2 + 4\cdot 0{,}1148^2 + 0{,}8852^2\right) + 0{,}8852^2\cdot 1{,}4\Big] + \frac{1}{2IE}\left[1{,}1148^2\cdot 1{,}4\right] = \frac{3{,}353}{IE}$$

$$\delta_{22} = \int \frac{m_2\cdot m_2}{IE}\,ds = \frac{1}{IE}\left[2\cdot 0{,}7^2\cdot 2 + \frac{0{,}7^2\cdot 1{,}4}{3}\right] + \frac{1}{2IE}\left[\frac{0{,}7^2\cdot 1{,}4}{3}\right] = \frac{2{,}303}{IE}$$

$$\delta_{33} = \int \frac{m_3\cdot m_3}{IE}\,ds = \frac{1}{IE}\left[2\cdot 1^2\cdot 2 + 1^2\cdot 1{,}4\right] + \frac{1}{2IE}\left[1^2\cdot 1{,}4\right] = \frac{6{,}1}{IE}$$



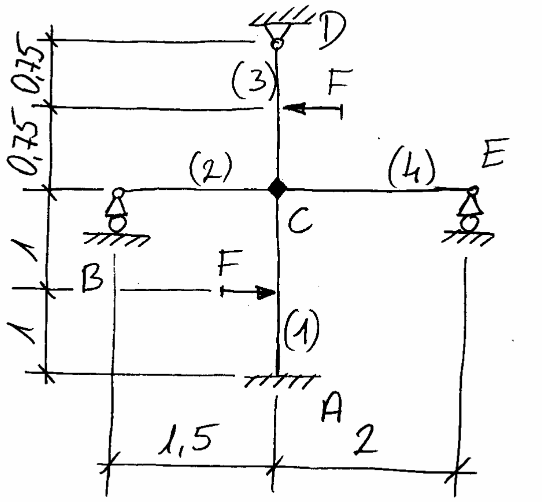
$$\begin{bmatrix} 3{,}353 & 0 & 0 \\ 0 & 2{,}303 & 0 \\ 0 & 0 & 6{,}1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 743{,}568 \\ -196 \\ -840 \end{bmatrix} \Rightarrow x_1 = -\frac{\delta_{10}}{\delta_{11}} = 221{,}76 \text{ N}$$

$$x_2 = -\frac{\delta_{20}}{\delta_{22}} = -85{,}1 \text{ N}$$

$$x_3 = -\frac{\delta_{30}}{\delta_{33}} = -137{,}7 \text{ Nm}$$

$$M = M_0 + X_1\cdot m_1 + X_2\cdot m_2 + X_3\, m_3$$

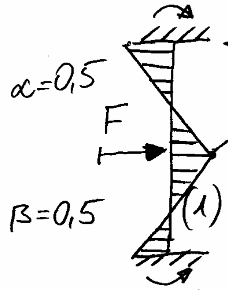
(4)



$I_1 = I_3 = I \qquad F = 4800\,N$

$I_2 = I_4 = 0{,}8 \cdot I \qquad AE = \infty$

$M = \frac{F \cdot \ell_1}{8} = \frac{4200 \cdot 2}{8} = 1200\,Nm = B_{10}^{(1)}$

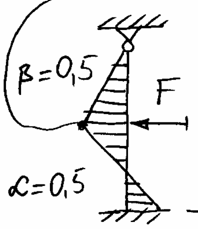


$F = \beta^2(1+2\alpha)F = 2400\,N = -B_{20}^{(1)}$

$\frac{F\ell_1}{8} = 1200\,Nm$

$M = \frac{F \cdot \ell_1}{8} = 1200\,Nm$

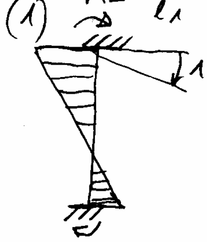
$M = 1125\,Nm$



$F = \frac{1}{2}(2 - 3\alpha^2 + \alpha^3)F = 3300\,N = B_{20}^{(3)}$

$M = \frac{\alpha}{2}(\beta + \beta^2) F \cdot \ell_3 = 1350\,Nm = B_{10}^{(3)}$

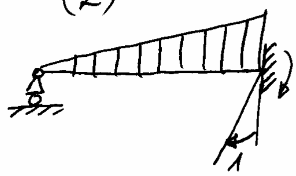
szabadságfokok



$M = \frac{4 I_1 E}{\ell_1} = 2IE = B_{11}^{(1)}$

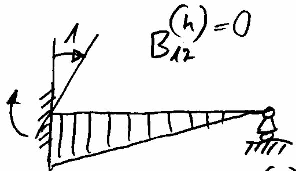
$F = \frac{6 I_1 E}{\ell_1^2} = 1{,}5IE = -B_{12}^{(1)}$

$M = \frac{2 I_1 E}{\ell_1} = 1 \cdot IE$



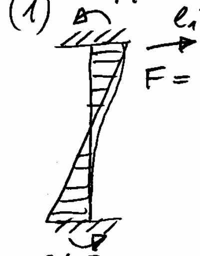
$M = \frac{3 I_2 E}{\ell_2} = 1{,}6IE = B_{11}^{(2)}$

$B_{12}^{(2)} = 0$



$B_{12}^{(4)} = 0$

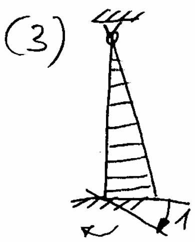
$M = \frac{3 I_4 E}{\ell_4} = 1{,}2IE = B_{11}^{(4)}$



$M = \frac{6 I_1 E}{\ell_1^2} = 1{,}5IE = -B_{21}^{(1)}$

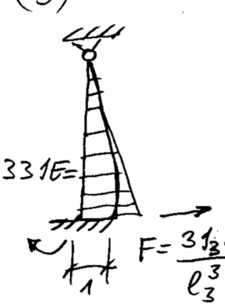
$F = \frac{12 I_1 E}{\ell_1^3} = 1{,}5IE = B_{22}^{(1)}$

$M = \frac{6 I_1 E}{\ell_1^2} = 1{,}5IE$



$F = \frac{3 I_3 E}{\ell_3^2} = 1{,}\overline{33}IE = B_{12}^{(3)}$

$M = \frac{3 I_3 E}{\ell_3} = 2IE = B_{11}^{(3)}$



$M = \frac{3 I_3 E}{\ell_3^2} = 1{,}33IE = B_{21}^{(3)}$

$F = \frac{3 I_3 E}{\ell_3^3} = 0{,}\overline{888}IE = B_{22}^{(3)}$

$B_{22}^{(2)} = B_{22}^{(4)} = 0$

$B_{21}^{(2)} = B_{21}^{(4)} = 0$

$B_{10} = 1200 + 1350 = 2550\,Nm$

$B_{20} = -2400 + 3300 = 900\,N$

$B_{11} = 2IE + 1{,}6IE + 2IE + 1{,}2IE = 6{,}8IE$

$B_{12} = B_{21} = -1{,}5IE + 0 + 1{,}\overline{33}IE + 0 = -0{,}1\overline{66}IE$

$B_{22} = 1{,}5IE + 0 + 0{,}\overline{888}IE + 0 = 2{,}3\overline{88}IE$

$$6{,}8\,1E \cdot \delta_1 - 0{,}1\overline{6}\,1E \cdot \delta_2 = -2550$$

$$-0{,}1\overline{6}\,1E \cdot \delta_1 + 2{,}3\overline{8}\,1E \cdot \delta_2 = -900$$

$$\delta_1 = -\frac{384{,}9}{1E} \quad rad$$

$$\delta_2 = -\frac{403{,}6}{1E} \quad m$$

$$M = M_0 + \delta_1 \cdot m_1 + \delta_2 \cdot m_2$$