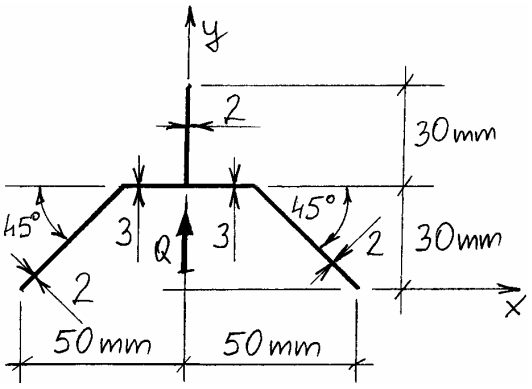
**1. Feladat** *(25 pont)*: A vázolt nyílt, vékonyfalú keresztmetszetet a Q nyíróerő terheli.

a) Határozza meg a vázolt szelvény M nyírási középpontjának helyét!

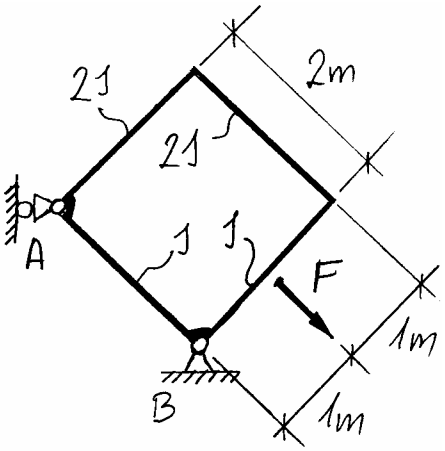
b) Határozza meg a keresztmetszet $\omega^*$ redukált cikkterület függvényét a jellemző értékek feltüntetésével!




**2. Feladat** *(25 pont)*: A vázolt zárt keretet az F erő terheli. Minden szakasz anyaga azonos (E).

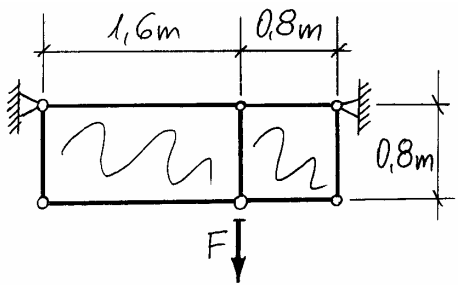
a) Határozza meg a keret hajlító-nyomaték igénybevételi ábráját <u>σ-ponti módszerrel</u>!

<u>Adatok:</u> $E = \text{áll.}\,;\ F = 5\,\text{kN}$




**3. Feladat** *(25 pont)*: A vázolt lemezzel merevített tartót az F erő terheli. Minden rúdja azonos anyagú (E) és azonos $A_k$ keresztmetszetű. Minden lemez azonos anyagú (G) és azonos „v” vastagságú.

a) Határozza meg a rudak normálerő-igénybevételi és a lemezek nyírófolyam-igénybevételi ábráit a jellemző értékek feltüntetésével!

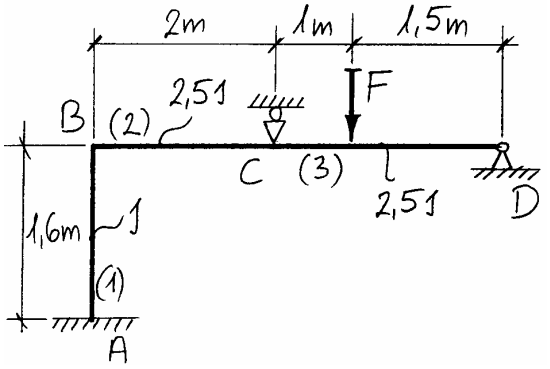


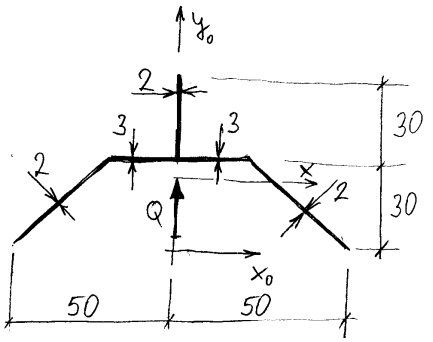

<u>Adatok:</u> $F = 3000\,\text{N}\,;\ E = 200\,\text{GPa} = 2,6 \cdot G\,;\ A = 400\,\text{mm}^2\,;\ v = 0,52\,\text{mm}$

**4. Feladat** *(25 pont)*: A vázolt törttengelyű tartó minden rúdja azonos anyagú (E=áll.), húzó-nyomó merevsége végtelen, a rudak másodrendű nyomatékai adottak és az F koncentrált erő terheli.

a) <u>Nyomaték osztási módszerrel</u> határozza meg a tartó hajlító igénybevételi ábráját!

<u>Adatok:</u> $F = 12\,\text{kN}\,;\ AE = \infty\,;\ E = \text{áll.}$






$\xi_M ; \eta_M = ?$ $\omega^* = ?$

$\tau$ dolgozik ?

$\xi_M = 0$ ← szimmetria

$\eta_M = \frac{1}{I_y} \int \omega \cdot x \cdot \nu \, dA$

$x_S = 0$ ← szimmetria

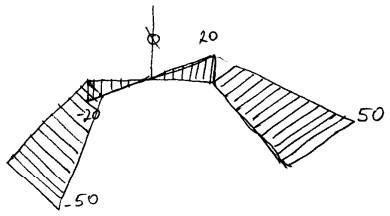
$S_x = 2 \cdot [30\sqrt{2} \cdot 2 \cdot 15] + 40 \cdot 3 \cdot 30 + 30 \cdot 2 \cdot 45 = 8845{,}58 \ mm^3$

$y_S = \frac{S_x}{A} = 25{,}294 \ mm$ $A = 2 \cdot [2 \cdot 30\sqrt{2}] + 40 \cdot 3 + 30 \cdot 2 = 349{,}706 \ mm^2$

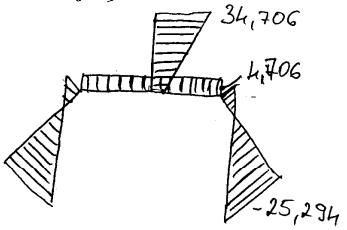
$I_y = 2 \cdot [2 \cdot 30\sqrt{2} \cdot (\frac{30^2}{12} + 35^2)] + 40 \cdot 3 \cdot (\frac{40^2}{12} + 0) + 2 \cdot 30 (0+0) = 236617{,}3 \ mm^4$

$I_x = 2 \cdot [2 \cdot 30\sqrt{2} \cdot (\frac{30^2}{12} + 10{,}294^2)] + 40 \cdot 3 (\frac{0^2}{12} + 4{,}706^2) + 2 \cdot 30 \cdot (\frac{30^2}{12} + 19{,}706^2) = 61168{,}2 \ mm^4$

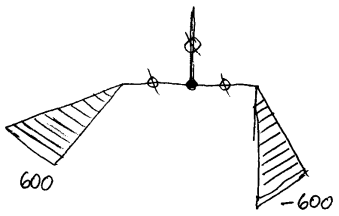
$x(u)$




$y(u)$




$\omega_p(u)$

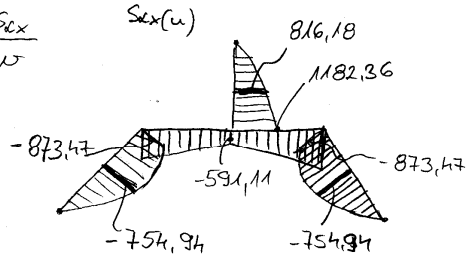



$\int \omega_p \cdot x \cdot \nu \, du = 2 \cdot [2 \cdot \frac{30\sqrt{2}}{6} (-20 \cdot 0 + 4 \cdot -35 \cdot 300 + 600 \cdot -50)] =$

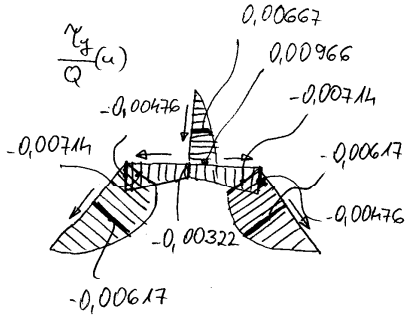
$= 2036467{,}53 \ mm^5$

$\eta_M = \frac{2036467{,}53}{236617{,}3} = 8{,}6066 \ mm$

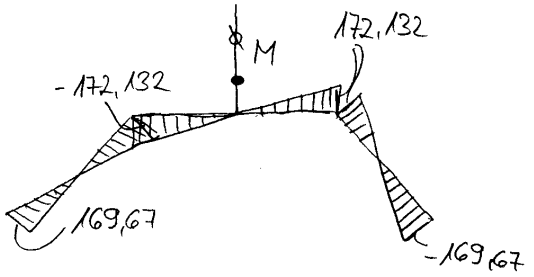
$\tau_y = \frac{Q \cdot S_{cx}}{I_x \cdot \nu}$ $S_{cx}(u)$

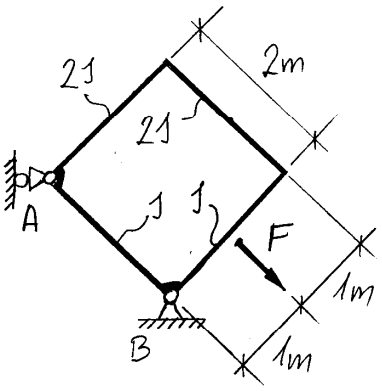



$\frac{\tau_y}{Q}(u)$




$\omega_0 = \omega^*$





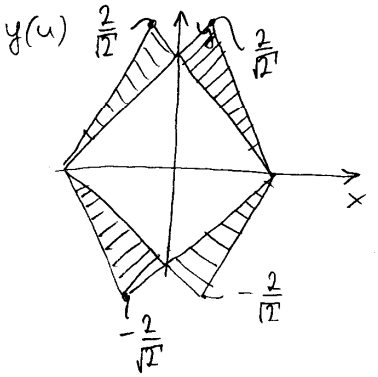

$F = 5\,kN \qquad E = all$

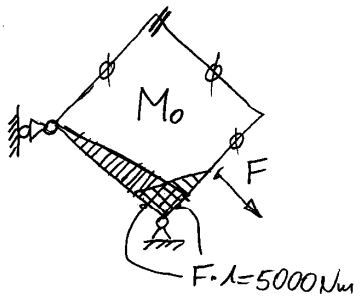
$$L_s = \frac{2}{1E} + \frac{2}{1E} + \frac{2}{2\,1E} + \frac{2}{2\,1E} = \frac{6}{1E}$$

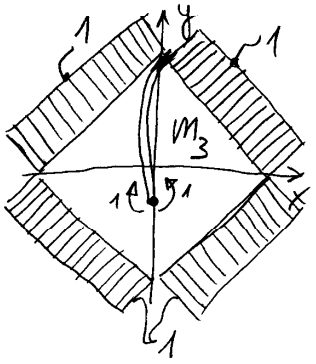
$$y_0 = \frac{\int \frac{y}{1E}\,du}{\int \frac{du}{1E}} = \frac{\int \frac{y}{1E}\,du}{L_s} = \frac{-\frac{\sqrt{2}}{6}}{6} = -0{,}2357\ m$$

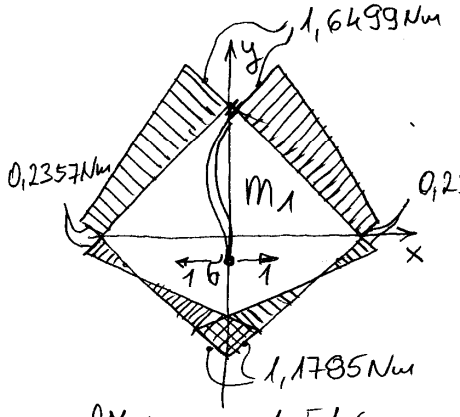
$$\int \frac{y}{1E}\,du = \frac{1}{1E}\left[\frac{-\sqrt{2}\cdot 2}{2}\right]\cdot 2 + \frac{1}{2\,1E}\left[\frac{\sqrt{2}\cdot 2}{2}\right]\cdot 2 =$$

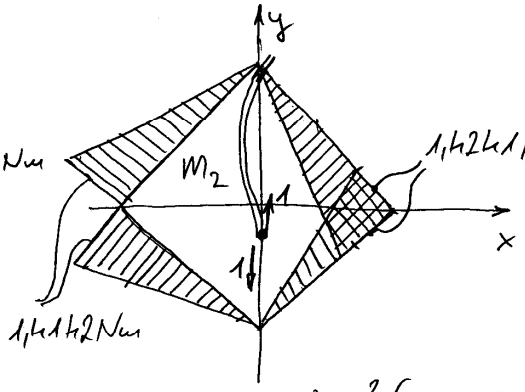
$$= -\frac{2\sqrt{2}}{1E} + \frac{\sqrt{2}}{1E} = \frac{-\sqrt{2}}{1E}$$




$$x_1 = -\frac{\delta_{10}}{\delta_{11}} = -1607{,}3$$

$$x_2 = -\frac{\delta_{20}}{\delta_{22}} = 445{,}04$$

$$x_3 = -\frac{\delta_{30}}{\delta_{33}} = 1250$$

$$\delta_{10} = \int \frac{M_0 \cdot m_1}{1E}\,du = \frac{1}{1E}\left[\frac{1}{6}\left(0 + 4\cdot 2500\cdot 0{,}82495 + 1{,}1785\cdot 5000\right) + \frac{2}{6}\left(1{,}1785\cdot 5000 + 4\cdot 2500\cdot 0{,}4714 + 0\right)\right] = \frac{1}{1E}\left[2357 + 3535{,}5\right] = \frac{5892{,}5}{1E}$$

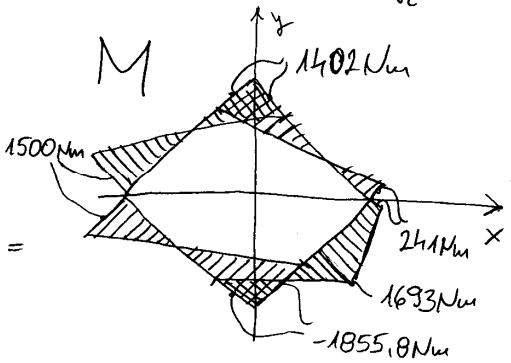
$$\delta_{20} = \int \frac{M_0 \cdot m_2}{1E}\,du = \frac{1}{1E}\left[\frac{1}{6}\left(0 + 4\cdot 2500\cdot 0{,}3560 + 0\right) + \frac{2}{6}\left(0 - 4\cdot 2500\cdot 0{,}71205 + 0\right)\right] = -\frac{1780{,}166}{1E}$$

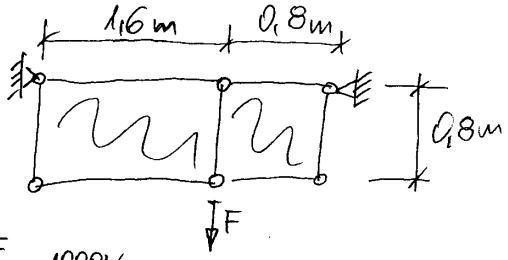
$$\delta_{30} = \int \frac{M_0 m_3}{1E}\,du = \frac{1}{1E}\left[\frac{-5000\cdot 1}{2}\cdot 1 - \frac{5000\cdot 2}{2}\cdot 1\right] = -\frac{7500}{1E}$$

$$\delta_{11} = \int \frac{m_1 \cdot m_1}{1E}\,du = \frac{1}{1E}\left[\frac{2}{6}\left(0{,}2357^2 + 4\cdot 0{,}4714^2 + 1{,}1785^2\right)\right]\cdot 2 + \frac{1}{2\,1E}\left[\frac{2}{6}\left(0{,}2357^2 + 4\cdot 0{,}9428^2 + 1{,}6499^2\right)\right]\cdot 2 = \frac{3{,}666}{1E}$$

$$\delta_{22} = \frac{1}{1E}\left[\frac{1{,}4142\cdot 2}{2}\cdot\frac{2}{3}\cdot 1{,}4142\right]\cdot 2 + \frac{1}{2\,1E}\left[\frac{1{,}4142\cdot 2}{2}\cdot\frac{2}{3}\cdot 1{,}4142\right]\cdot 2 = \frac{4}{1E}$$

$$\delta_{33} = \frac{1}{1E}\left[2\cdot 1\cdot 1\right]\cdot 2 + \frac{1}{2\,1E}\left[2\cdot 1\cdot 1\right]\cdot 2 = \frac{6}{1E}$$



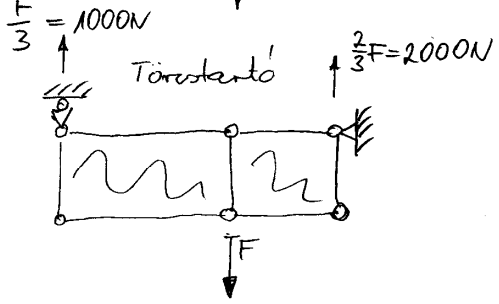


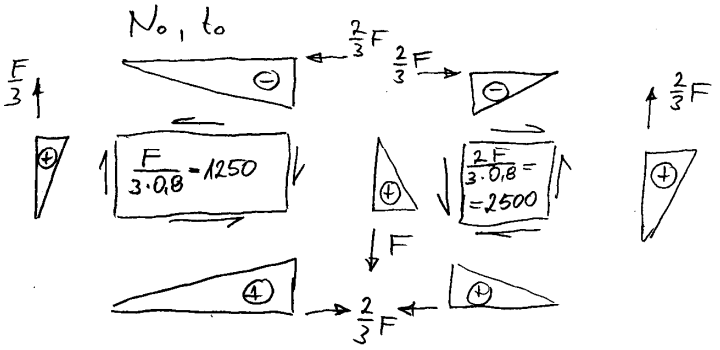

$E = 200\,GPa = 2,6G$

$A = 400\,mm^2$

$v = 0,52\,mm$

$F = 3000\,N$

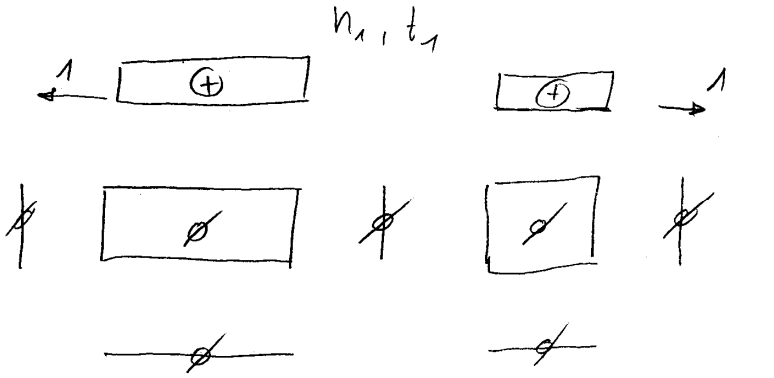



$$\delta_{10} = \int \frac{N_0 \cdot n_1}{AE} ds + \int \frac{t_0 \cdot t_1}{G \cdot v} \cdot dA =$$

$$= \frac{1}{AE}\left[-\frac{2}{3}F \cdot 1,6 \cdot 1 - \frac{2F \cdot 0,8}{3 \cdot 2} \cdot 1\right] +$$

$$+ \frac{1}{G \cdot v}[0] = \frac{-0,8F}{AE} = -\frac{2400}{AE}$$

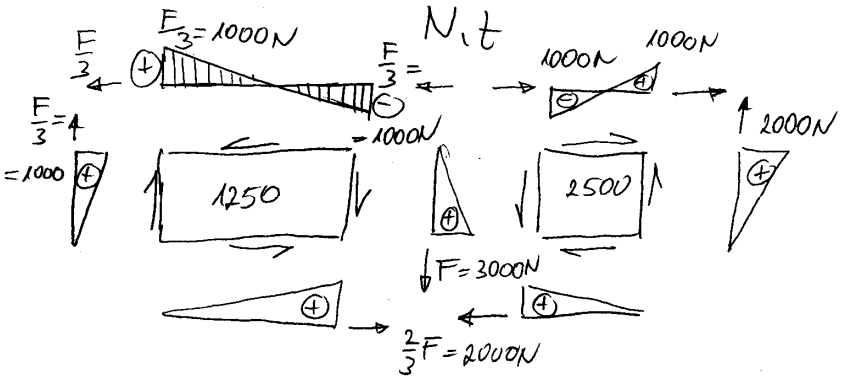



$$\delta_{11} = \int \frac{n_1 \cdot n_1}{AE} ds + \int \frac{t_1 \cdot t_1}{G \cdot v} dA =$$

$$= \frac{1}{AE}\left[1^2 \cdot 1,6 + 1^2 \cdot 0,8\right] + \frac{1}{G \cdot v}[0] =$$

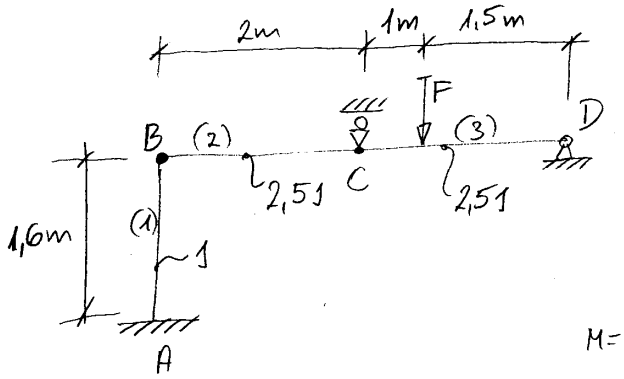
$$= \frac{2,4}{AE}$$

$$X_1 = -\frac{\delta_{10}}{\delta_{11}} = \frac{0,8F}{2,4} = \frac{F}{3} = 1000N$$




$N = N_0 + X_1 \cdot n_1$

$t = t_0 + X_1 \cdot n_1$




$F = 12\,kN \qquad AE = \infty$

$M = ? \qquad E = all.$

$F = 9504N = 0{,}792F$

$\alpha = 0{,}4 \qquad \beta = 0{,}6$

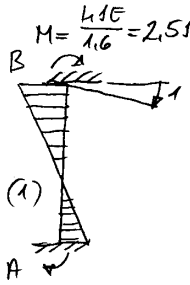
$M = 3744\,Nm$

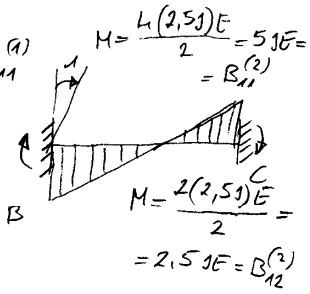
$F = 2496N = 0{,}208N$

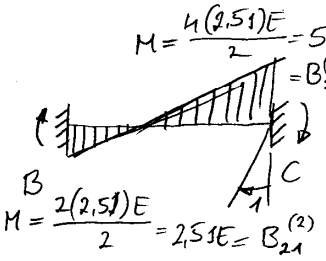
$M = \frac{\alpha}{2}(\beta+\beta^2)Fl = 5760\,Nm = -B_{20}$

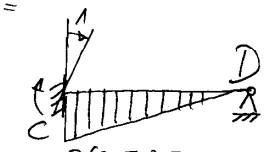
$B_{10} = 0\,Nm$

| A | B | | C | |
|---|---|---|---|---|
| 1 | 1 | 2 | 2 | 3 |
| 1 | 1/3 | 2/3 | 5/8 | 3/8 |
| | | | | -5760 |
| | | +1800 | +3600 | +2160 |
| -300 | -600 | -1200 | -600 | |
| | | +188 | +375 | +225 |
| -31 | -62 | -126 | -63 | |
| | | +20 | +40 | +23 |
| -3 | -6 | -14 | -7 | |
| | | +2 | +4 | +3 |
| 0 | 0 | -2 | -1 | |
| | | | 1 | 0 |
| -334 | -668 | 668 | 3349 | -3349 |




$B_{10} = 0\,Nm \qquad B_{11} = 2{,}51E + 5{,}1E = 7{,}51E$

$B_{20} = -5760\,Nm \qquad B_{12} = B_{21} = 2{,}51E$

$B_{22} = 5{,}1E + 3E = 8{,}1E$

$$\left.\begin{aligned} 7{,}51E\cdot\sigma_1 + 2{,}51E\sigma_2 &= 0 \\ 2{,}51E\sigma_1 + 8{,}1E\sigma_2 &= 5760 \end{aligned}\right\} \quad \sigma_1 = -\frac{267{,}91}{1E} \qquad \sigma_2 = \frac{803{,}72}{1E}$$