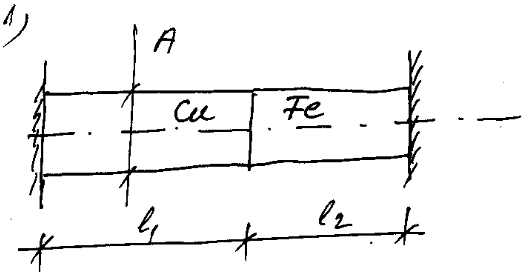
1)



A rudak végei $t = 0°C$ hőmérsékleten feszültségmentesen illeszkednek.
Határozza meg
- Az egyes rudakban ébredő feszültséget.
- A két rúd érintkezési pontjának elmozdulását

ha a hőmérséklet 50°C-ra emelkedik.

$l_1 = 15$ cm

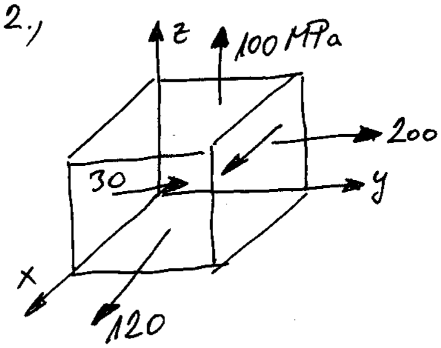
$l_2 = 10$ cm

$A = 4$ cm^2

$\alpha_{Cu} = 16 \cdot 10^{-6} \frac{1}{°C}$ $E_{Cu} = 130$ GPa

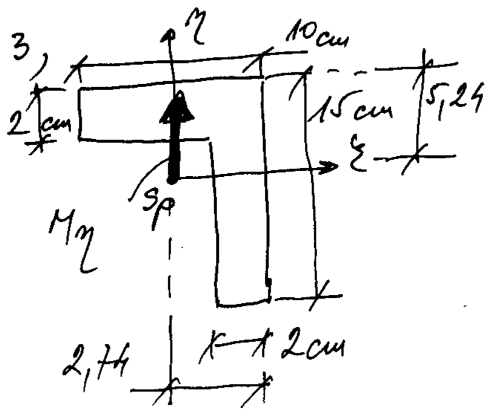
$\alpha_{Fe} = 12 \cdot 10^{-6} \frac{1}{°C}$ $E_{Fe} = 210$ GPa

2.)



Határozza meg a főfeszültségeket és a σ_3 főfeszültséghez tartozó főirány egységvektorát.

3)

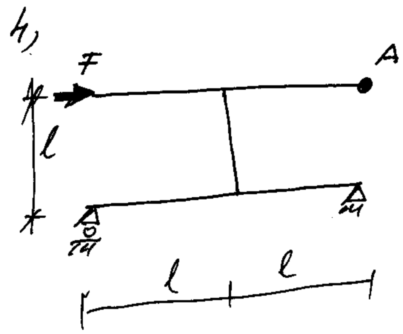


Határozza meg a semleges szál helyét és a maximális feszültséget.

$M_\eta = 10$ kNm

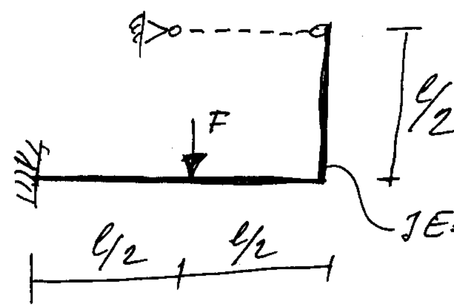
$$\underline{\underline{J}}_S = \begin{bmatrix} 1008 & 339 \\ 339 & 356 \end{bmatrix} \text{cm}^4$$

4)



Határozza meg az A pont függőleges irányú elmozdulását.
A tartó állandó merevségű.
IE=állandó

5)



IE=állandó

Rajzolja meg a tartó nyomatéki ábráját.
A kötél nem nyúlik meg.
Csak a hajlításból származó belső munkát vegye figyelembe.

(1)

$$\left(\lambda_{t_1}+\lambda_1\right)+\left(\lambda_{t_2}+\lambda_2\right)=0$$

$$\alpha_{Cu}\,\ell_1\,\Delta t+\frac{N\,\ell_1}{A\,E_{Cu}}+\alpha_{Fe}\,\ell_2\,\Delta t+\frac{N\,\ell_2}{A\,E_{Fe}}=0$$

$$A\left(\alpha_{Cu}\ell_1+\alpha_{Fe}\ell_2\right)\Delta t+N\left(\frac{\ell_1}{E_{Cu}}+\frac{\ell_2}{E_{Fe}}\right)=0$$

$$N=\frac{-A\left(\alpha_{Cu}\ell_1+\alpha_{Fe}\ell_2\right)\Delta t}{\frac{\ell_1}{E_{Cu}}+\frac{\ell_2}{E_{Fe}}}=$$

$$=\frac{-400\left(16\cdot10^{-6}\cdot150+12\cdot10^{-6}\cdot100\right)\cdot50}{\frac{150}{130\cdot10^3}+\frac{100}{210\cdot10^3}}=-44\,171\ \mathrm{N}$$

$$\boxed{\sigma_1=\sigma_2=\frac{N}{A}=\frac{-44\,171}{400}=-110{,}4\ \mathrm{MPa}}$$

$$\boxed{\Delta x}=\lambda_{t_1}+\lambda_1=\alpha_{Cu}\,\ell_1\,\Delta t+\frac{N\,\ell_1}{A\,E_{Cu}}=$$

$$=16\cdot10^{-6}\cdot150\cdot50+\frac{-44\,171\cdot150}{400\cdot130\cdot10^3}=0{,}12-0{,}1274=$$

$$=\boxed{-0{,}0074\ \mathrm{mm}\ (\leftarrow)}$$

2

$$\underline{\underline{F}} = \begin{bmatrix} 120 & 30 & 0 \\ 30 & 200 & 0 \\ 0 & 0 & 100 \end{bmatrix} \text{MPa}$$

$$\det\left(\underline{\underline{F}} - \sigma_i \underline{\underline{E}}\right) = \begin{vmatrix} 120-\sigma_i & 30 & 0 \\ 30 & 200-\sigma_i & 0 \\ 0 & 0 & 100-\sigma_i \end{vmatrix} = (100-\sigma_i)\left[(120-\sigma_i)(200-\sigma_i) - 30^2\right]$$

$$\sigma_i = 100 \text{ MPa}$$

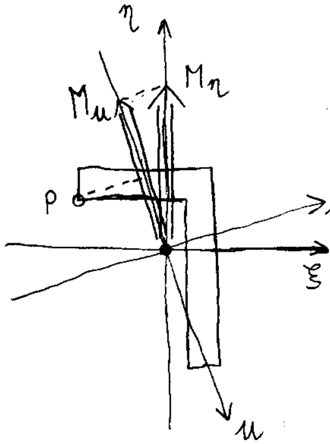
$$\sigma_i^2 - 320\,\sigma_i + 23100 = 0$$

$$\sigma_i = \frac{320 \pm \sqrt{320^2 - 4 \cdot 23100}}{2} = 160 \pm 50 = \begin{cases} 210 \\ 110 \end{cases}$$

$$\boxed{\begin{aligned} \sigma_1 &= 210 \text{ MPa} \\ \sigma_2 &= 110 \text{ MPa} \\ \sigma_3 &= 100 \text{ MPa} \end{aligned}}$$

$$\sigma_3 = \sigma_z \longrightarrow \boxed{\underline{n}_3} = \underline{n}_z = \pm\underline{k} = \boxed{\pm\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}$$

3



$$\underline{e}_u^T \,\underline{\underline{I}}_{(\xi,\eta)}\, \underline{v}_{M\perp} = \begin{bmatrix}\cos\beta & \sin\beta\end{bmatrix}\begin{bmatrix}1008 & 339\\ 339 & 356\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = 0$$

$$\begin{bmatrix}\cos\beta & \sin\beta\end{bmatrix}\begin{bmatrix}1008\\ 339\end{bmatrix} = 1008\cos\beta + 339\sin\beta = 0$$

$$\mathrm{tg}\,\beta = \frac{-1008}{339} = -2{,}973 \longrightarrow \boxed{\beta = -71{,}41^0}$$

$$\underline{e}_u = \begin{bmatrix}\cos\beta\\ \sin\beta\end{bmatrix} = \begin{bmatrix}+0{,}3188\\ -0{,}9478\end{bmatrix}$$

$$\underline{e}_v = \begin{bmatrix}+0{,}9478\\ +0{,}3188\end{bmatrix}$$

$$M_u = -M_\eta \sin|\beta| = 10000 \sin 71{,}41^0 = -9478 \text{ Nm}$$

$$I_u = \underline{e}_u^T \,\underline{\underline{I}}_{(\xi,\eta)}\, \underline{e}_u = \begin{bmatrix}0{,}3188 & -0{,}9478\end{bmatrix}\begin{bmatrix}1008 & 339\\ 339 & 356\end{bmatrix}\begin{bmatrix}0{,}3188\\ -0{,}9478\end{bmatrix} =$$

$$= \begin{bmatrix}0{,}3188 & -0{,}9478\end{bmatrix}\begin{bmatrix}0\\ -229{,}3\end{bmatrix} = 217{,}3 \text{ cm}^4$$

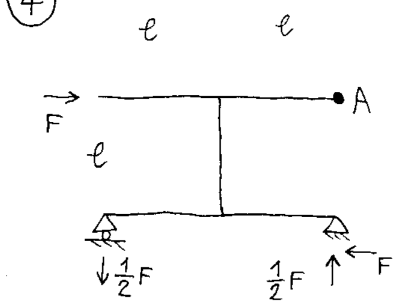
$$v_A = \underline{e}_v^T \,\underline{r}_A = \begin{bmatrix}+0{,}9478 & +0{,}3188\end{bmatrix}\begin{bmatrix}-72{,}6\\ +32{,}4\end{bmatrix} = -58{,}48 \text{ mm}$$

$$v_B = \underline{e}_v^T \,\underline{r}_B = \begin{bmatrix}+0{,}9478 & +0{,}3188\end{bmatrix}\begin{bmatrix}+27{,}4\\ +52{,}4\end{bmatrix} = +42{,}67 \text{ mm}$$

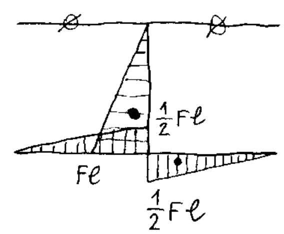
$$|v_A| > |v_B| \longrightarrow |\sigma_A| > |\sigma_B|$$

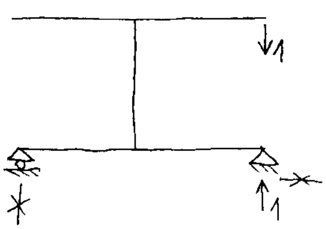
$$\boxed{\sigma_A} = \frac{M_u}{I_u} v_A = \frac{-9478\cdot 10^3}{217{,}3\cdot 10^4}(-58{,}48) = \boxed{+255{,}1 \text{ MPa} = \sigma_{max}}$$

4

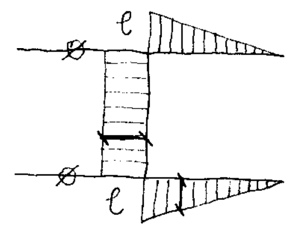


M





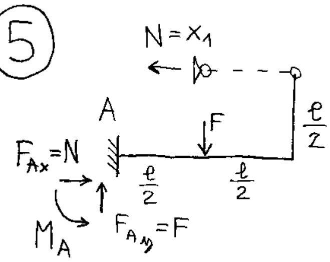
m



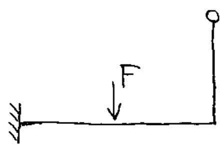
$$\boxed{f_y^{(A)}} = \int \frac{M\, m}{IE}\, ds = \frac{1}{IE}\left[+\frac{1}{2}\cdot \ell \cdot F\ell\,(\ell) + \frac{1}{2}\,\ell\cdot\frac{1}{2}F\ell\left(\frac{2}{3}\ell\right)\right] =$$

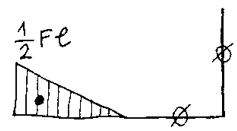
$$= \frac{F\ell^3}{IE}\left(\frac{1}{2}+\frac{1}{6}\right) = \boxed{\frac{2}{3}\,\frac{F\ell^3}{IE}\ (\downarrow)}$$

5

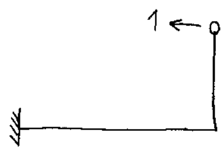


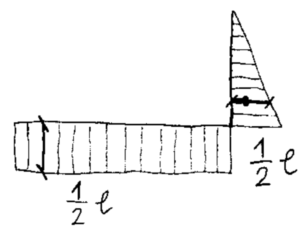
M_0





m_1

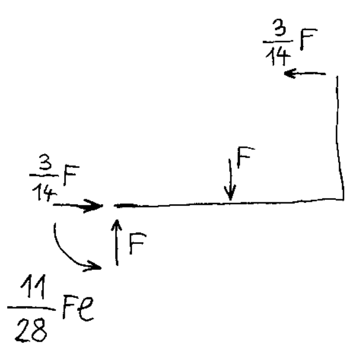




$$\delta_{10} = \frac{1}{IE}\left[\frac{-1}{2}\cdot\frac{\ell}{2}\cdot\frac{1}{2}F\ell\left(\frac{1}{2}\ell\right)\right] = \frac{-1}{16}\,\frac{F\ell^3}{IE}$$

$$\delta_{11} = \frac{1}{IE}\left[\ell\cdot\frac{1}{2}\ell\left(\frac{1}{2}\ell\right)+\frac{1}{2}\cdot\frac{\ell}{2}\cdot\frac{1}{2}\ell\left(\frac{\cancel{2}}{3}\cdot\frac{1}{\cancel{2}}\ell\right)\right] = \frac{\ell^3}{IE}\left(\frac{1}{4}+\frac{1}{24}\right) = \frac{7}{24}\,\frac{\ell^3}{IE}$$

$$\boxed{N = x_1 = \frac{-\delta_{10}}{\delta_{11}} = \frac{\frac{1}{16}\,\frac{F\ell^3}{IE}}{\frac{7}{24}\,\frac{\ell^3}{IE}} = \frac{3}{14}\,F\ (\leftarrow)}$$



M