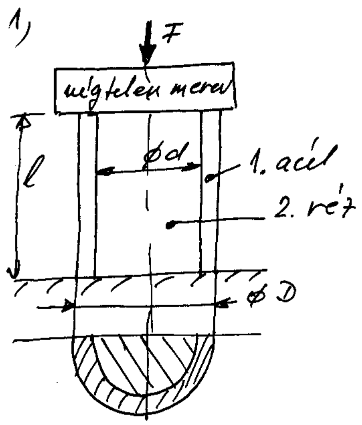
1)



Sárgaréz belső magot (2) acélköpeny (1.) veszi körül.

D = 200 mm   d = 120 mm

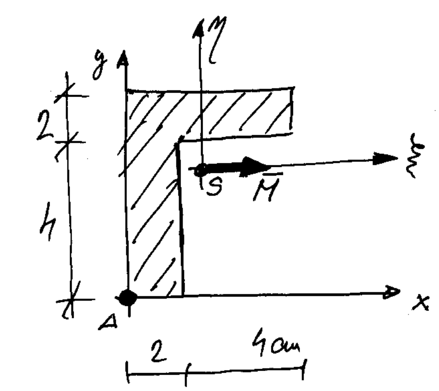
l = 500 mm

E_1 = 210 GPa E_2 = 90 GPa

Határozza meg az acélban (σ_1) és a rézben (σ_2) ébredő feszültséget. Határozza meg a végtelen merev lap elmozdulását.

F = 700 kN

2)



Határozza meg a semleges szál helyét és az "A" pontban ébredő feszültség értékét.

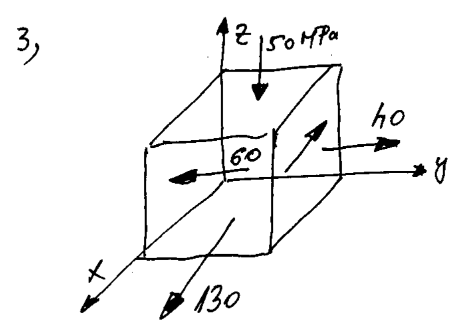
$x_S = 2,2$ cm $J_\xi = 57,9$ cm^4

$y_S = 3,8$ cm $J_\eta = 57,9$ cm^4

$J_{\xi\eta} = 28,8$ cm^4

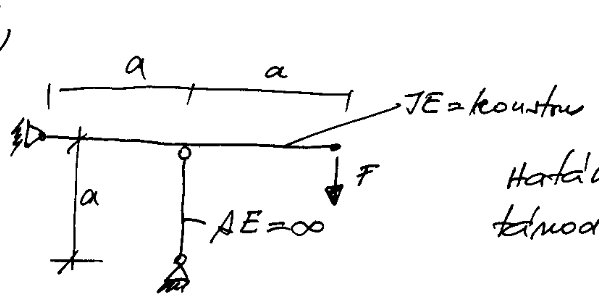
$M = 1000$ Nm

3)



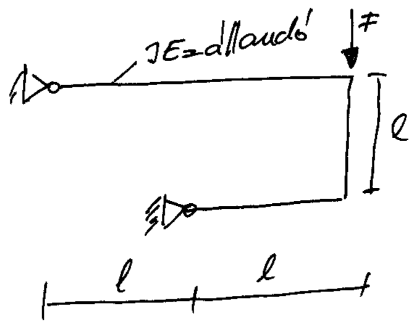
Határozza meg az ábrázolt feszültségi állapot főfeszültségeinek nagyságát és a σ_1 főfeszültséghez tartozó főirány egységvektorát.

4)



Határozza meg a F erő támadáspontjának szögelfordulását.

5)



Rajzolja meg a tartó nyomatéki ábráját.
A belső munka számításánál csak a hajlítónyomatékot vegye figyelembe.

1

1.) $\sum F_y = 0 = F_1 + F_2 - F$

2.) $\lambda_1 = \lambda_2 = \lambda$

$$\frac{F_1 \ell}{A_1 E_1} = \frac{F_2 \ell}{A_2 E_2}$$

$$A_1 = \frac{(D^2 - d^2)\pi}{4} = \frac{(200^2 - 120^2)\pi}{4} = 20\,106\ mm^2$$

$$A_2 = \frac{d^2 \pi}{4} = \frac{120^2 \pi}{4} = 11\,310\ mm^2$$

1.) $F_2 = F - F_1$

$1 \rightarrow 2.)$

$$\frac{F_1}{A_1 E_1} = \frac{F - F_1}{A_2 E_2}$$

$$F_1 = \frac{\frac{F}{A_2 E_2}}{\frac{1}{A_1 E_1} + \frac{1}{A_2 E_2}} = \frac{F}{\frac{A_2}{A_1}\frac{E_2}{E_1} + 1} = \frac{700}{\frac{11310}{20106}\frac{90}{210} + 1} = 564\ kN$$

1.) $F_2 = 700 - 564 = 136\ kN$

$$\boxed{\sigma_1 = \frac{F_1}{A_1} = \frac{564 \cdot 10^3}{20\,106} = 28{,}05\ MPa}$$

$$\boxed{\sigma_2 = \frac{F_2}{A_2} = \frac{136 \cdot 10^3}{11\,310} = 12{,}02\ MPa}$$

2.) $$\boxed{\lambda = \lambda_1 = \frac{F_1 \ell}{A_1 E_1} = \frac{564 \cdot 10^3 \cdot 500}{20\,106 \cdot 210 \cdot 10^3} = 0{,}06679\ mm}$$

Megjegyzés:

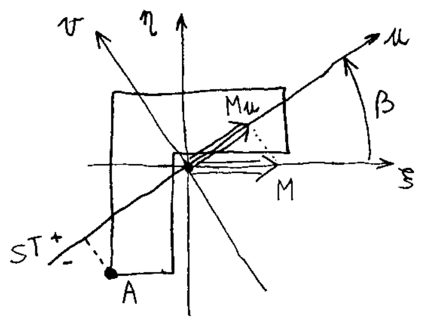
A javításnál a teljes körnek és a félkörnek tekintett keresztmetszetek esetén adódó megoldást is elfogadtuk.

2

$$\underline{e}_u \underline{\underline{I}}_{(\xi,\eta)} \underline{v}_{M\perp} = \begin{bmatrix} \cos\beta & \sin\beta \end{bmatrix} \begin{bmatrix} 57{,}9 & -28{,}8 \\ -28{,}8 & 57{,}9 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} \cos\beta & \sin\beta \end{bmatrix} \begin{bmatrix} -28{,}8 \\ 57{,}9 \end{bmatrix} = -28{,}8\cos\beta + 57{,}9 \sin\beta = 0$$

$$\operatorname{tg}\beta = \frac{28{,}8}{57{,}9} = 0{,}4974 \rightarrow \boxed{\beta = 26{,}45^\circ}$$



$$\underline{e}_u = \begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix} = \begin{bmatrix} 0{,}8953 \\ 0{,}4454 \end{bmatrix}$$

$$\underline{e}_v = \begin{bmatrix} -0{,}4454 \\ 0{,}8953 \end{bmatrix}$$

$$M_u = M\cos\beta = 10^3 \cos 26{,}45^\circ = 895{,}3 \text{ Nm}$$

$$I_u = \underline{e}_u^T \underline{\underline{I}}_{(\xi,\eta)} \underline{e}_u = \begin{bmatrix} 0{,}8953 & 0{,}4454 \end{bmatrix} \begin{bmatrix} 57{,}9 & -28{,}8 \\ -28{,}8 & 57{,}9 \end{bmatrix} \begin{bmatrix} 0{,}8953 \\ 0{,}4454 \end{bmatrix} =$$

$$= \begin{bmatrix} 0{,}8953 & 0{,}4454 \end{bmatrix} \begin{bmatrix} 39{,}01 \\ 0 \end{bmatrix} = 34{,}93 \text{ cm}^4$$

$$v_A = \underline{e}_v^T \underline{r}_A = \begin{bmatrix} -0{,}4454 & 0{,}8953 \end{bmatrix} \begin{bmatrix} -22 \\ -38 \end{bmatrix} = -24{,}22 \text{ mm}$$

$$\boxed{\sigma_A = \frac{M_u}{I_u} v_A = \frac{895{,}3 \cdot 10^3}{34{,}93 \cdot 10^4} (-24{,}22) = -62{,}08 \text{ MPa}}$$

3

$$\underline{\underline{F}} = \begin{bmatrix} 130 & -60 & 0 \\ -60 & 40 & 0 \\ 0 & 0 & -50 \end{bmatrix} \text{MPa}$$

$$\det(\underline{\underline{F}} - \sigma_i \underline{\underline{E}}) = \begin{vmatrix} 130-\sigma_i & -60 & 0 \\ -60 & 40-\sigma_i & 0 \\ 0 & 0 & -50-\sigma_i \end{vmatrix} =$$

$$= (-50-\sigma_i)\left[(130-\sigma_i)(40-\sigma_i) - 60^2\right] = 0$$

$$\downarrow$$

$$\sigma_i = -50$$

$$\sigma_i^2 - 170\sigma_i + 1600 = 0$$

$$\sigma_i = \frac{170 \pm \sqrt{170^2 - 4\cdot 1600}}{2} = 85 \pm 75 = \begin{cases} 160 \\ 10 \end{cases}$$

$$\boxed{\begin{aligned} \sigma_1 &= 160 \text{ MPa} \\ \sigma_2 &= 10 \text{ MPa} \\ \sigma_3 &= -50 \text{ MPa} \end{aligned}}$$

$$(\underline{\underline{F}} - \sigma_1 \underline{\underline{E}})\underline{n}'_1 = \underline{0}$$

$$\begin{bmatrix} 130-160 & -60 & 0 \\ -60 & 40-160 & 0 \\ 0 & 0 & -50 \end{bmatrix} \begin{bmatrix} n'_{1x} \\ n'_{1y} \\ n'_{1z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

1.) $-30 n'_{1x} - 60 n'_{1y} = 0$

2.) $-60 n'_{1x} - 120 n'_{1y} = 0$

3.) $-50 n'_{1z} = 0$

3.) $n'_{1z} = 0$

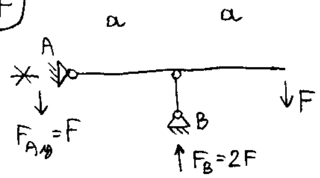
$n'_{1y} := 1$

1.) $n'_{1x} = \frac{-60}{30} n'_{1y} = -2$

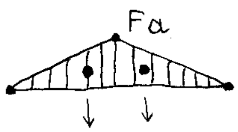
$$\underline{n}'_1 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \qquad |\underline{n}'_1| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

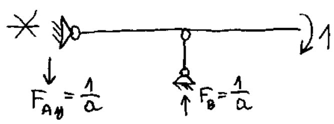
$$\boxed{\underline{n}_1 = \frac{\pm \underline{n}'_1}{|\underline{n}'_1|} = \frac{\pm 1}{\sqrt{5}} \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} = \pm \begin{bmatrix} -0{,}8944 \\ 0{,}4472 \\ 0 \end{bmatrix}}$$

4

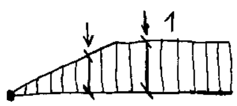


M



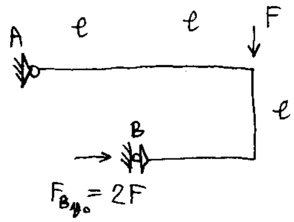


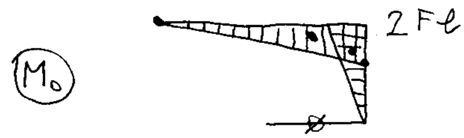
m

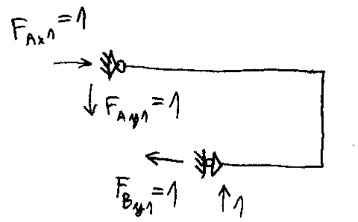


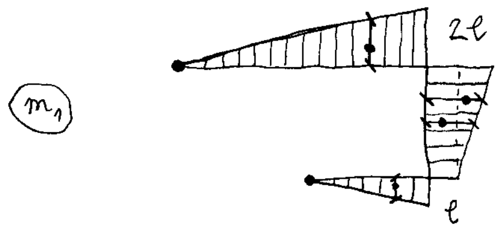
$$\boxed{\varphi=\int_{(l)}\frac{M\,m}{IE}\,ds=\frac{1}{IE}\left[\frac{1}{\cancel{2}}\,a\,Fa\left(\frac{\cancel{2}}{3}\cdot 1\right)+\frac{1}{2}\,a\,Fa\,(1)\right]=\frac{5}{6}\,\frac{Fa^2}{IE}}$$

5









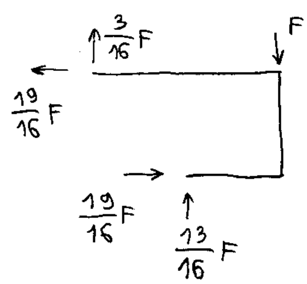
$$\delta_{10} = \int_{(\ell)} \frac{M_0 m_1}{IE}\, ds = \frac{1}{IE}\left[-\frac{1}{\cancel{2}} \cdot \cancel{2}\ell \cdot 2F\ell \left(\frac{2}{3} \cdot 2\ell\right) - \frac{1}{\cancel{2}} \ell \cdot \cancel{2}F\ell \left(\ell + \frac{2}{3}\ell\right)\right] =$$

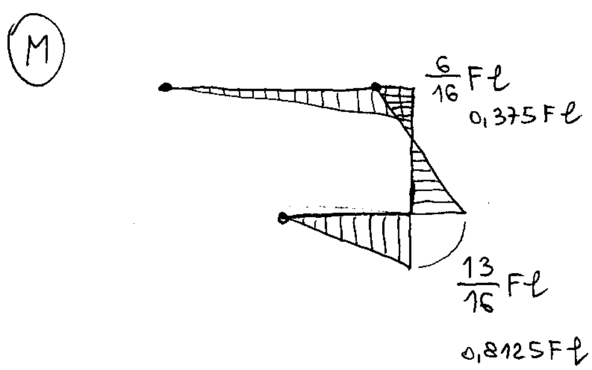
$$= \frac{-F\ell^3}{IE}\left(\frac{8}{3} + \frac{5}{3}\right) = \frac{-13}{3}\, \frac{F\ell^3}{IE}$$

$$\delta_{11} = \int_{(\ell)} \frac{m_1^2}{IE}\, ds = \frac{1}{IE}\left[\frac{1}{\cancel{2}} \cdot \cancel{2}\ell \cdot 2\ell \left(\frac{2}{3} \cdot 2\ell\right) + \ell \cdot \ell \left(\frac{3}{2}\ell\right) + \frac{1}{2} \cdot \ell \cdot \ell \left(\frac{5}{3}\ell\right) + \frac{1}{\cancel{2}} \ell \cdot \ell \left(\frac{\cancel{2}}{3}\ell\right)\right] =$$

$$= \frac{\ell^3}{IE}\left(\frac{8}{3} + \frac{3}{2} + \frac{5}{6} + \frac{1}{3}\right) = \frac{16}{3}\, \frac{\ell^3}{IE}$$

$$F_{Bx} = x_1 = \frac{-\delta_{10}}{\delta_{11}} = \frac{\frac{13}{3}\,\frac{F\ell^3}{IE}}{\frac{16}{3}\,\frac{\ell^3}{IE}} = \frac{13}{16} F$$





$$\sum M_A = 0 = -F \cdot 2\ell + \frac{13}{16} F\ell + F_{Bx}\ell$$

$$F_{Bx} = \frac{32-13}{16} F = \frac{19}{16} F \; (\rightarrow)$$