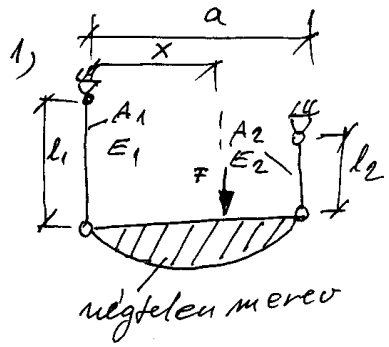
1)



Egy végtelen merev lapot két végén felfüggesztünk.

Hol helyezzük el az F erőt, hogy a lap vízszintes maradjon?

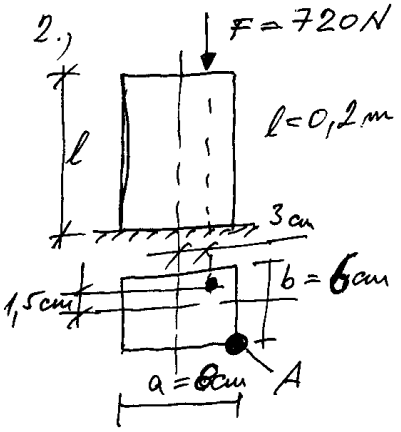
$A_1 = 2\,cm^2$ $A_2 = 3\,cm^2$

$E_1 = E_2 = 2 \cdot 10^5\,MPa$

$l_1 = 1\,m$ $l_2 = 0{,}5\,m$

$a = 4\,m$

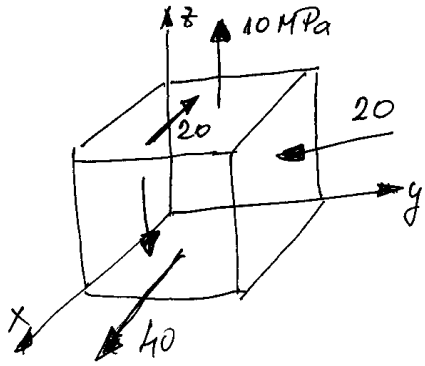
2.)



Egy rövid, tömör hasábot külpontosan terhel az F erő.

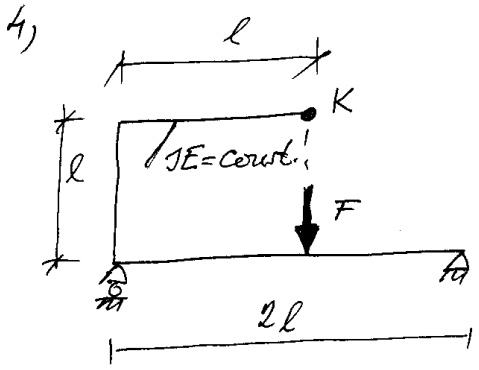
Határozza meg a Rajzolt "A" pontban ébredő feszültséget!

3)



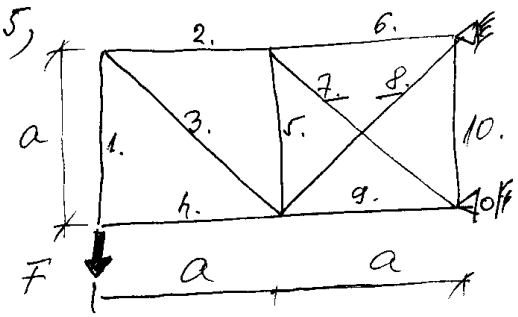
Határozza meg az ábrázolt feszültségi állapot főfeszültségeinek nagyságát és a σ_2 főfeszültséghez tartozó főirány egységvektorát.

4)



Határozza meg a K keresztmetszet függőleges irányú elmozdulását.

5)



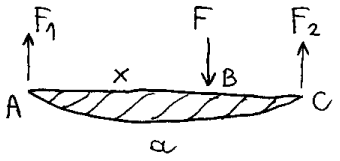
Határozza meg az egyes rudakban ébredő erőket.

(1)

$$\lambda_1 = \lambda_2$$

$$\frac{F_1 \ell_1}{A_1 E} = \frac{F_2 \ell_2}{A_2 E}$$

$$\frac{F_2}{F_1} = \frac{A_2}{A_1} \frac{\ell_1}{\ell_2} = \frac{3}{2} \frac{1}{0{,}5} = 3$$



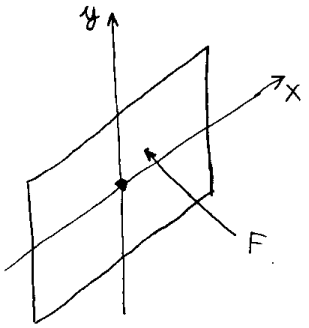
$$\Sigma M_B = 0 = -F_1 x + F_2 (a - x)$$

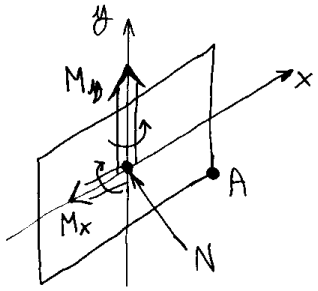
$$\frac{F_2}{F_1} = \frac{x}{a - x} = 3$$

$$x = 3a - 3x$$

$$\boxed{x = \frac{3}{4} a = \frac{3}{4} \cdot 4 = 3\,m}$$

(2)





$N = -F = -72\ kN$

$A = 80 \cdot 60 = 4800\ mm^2$

$M_x = -72 \cdot 10^3 \cdot 15 \cdot 10^{-3} = -1080\ Nm$

$M_y = +72 \cdot 10^3 \cdot 30 \cdot 10^{-3} = +2160\ Nm$

$I_x = \frac{60^3 \cdot 80}{12} = 1{,}44 \cdot 10^6\ mm^4$

$I_y = \frac{80^3 \cdot 60}{12} = 2{,}56 \cdot 10^6\ mm^4$

$$\sigma_A = \frac{N}{A} + \frac{M_x}{I_x} y_A - \frac{M_y}{I_y} x_A$$

$$\sigma_A = \frac{-72 \cdot 10^3}{4800} + \frac{-1080 \cdot 10^3}{1{,}44 \cdot 10^6}(-30) - \frac{+2160 \cdot 10^3}{2{,}56 \cdot 10^6}(+40)$$

$$\boxed{\sigma_A = -15 + 22{,}5 - 33{,}75 = -26{,}25\ MPa}$$

③

$$\underline{\underline{F}} = \begin{bmatrix} 40 & 0 & -20 \\ 0 & -20 & 0 \\ -20 & 0 & 10 \end{bmatrix} \text{MPa}$$

$$\det\left(\underline{\underline{F}} - \sigma_i \underline{\underline{E}}\right) = \begin{vmatrix} 40-\sigma_i & 0 & -20 \\ 0 & -20-\sigma_i & \\ -20 & 0 & 10-\sigma_i \end{vmatrix} =$$

$$= (-20-\sigma_i)\left[(40-\sigma_i)(10-\sigma_i) - 20^2\right] = 0$$

$$\downarrow$$

$$\sigma_i = -20 \text{ MPa}$$

$$\sigma_i^2 - 50\sigma_i = 0$$

$$\sigma_i(\sigma_i - 50) = 0 \rightarrow \sigma_i = \begin{cases} 50 \\ 0 \end{cases}$$

$\sigma_1 = 50$ MPa

$\sigma_2 = 0$

$\sigma_3 = -20$ MPa

$$\left(\underline{\underline{F}} - \sigma_2 \underline{\underline{E}}\right)\underline{n}'_2 = \underline{0}$$

$$\begin{bmatrix} 40 & 0 & -20 \\ 0 & -20 & 0 \\ -20 & 0 & 10 \end{bmatrix}\begin{bmatrix} n'_{2x} \\ n'_{2y} \\ n'_{2z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

1.) $40 n'_{2x} - 20 n'_{2z} = 0$

2.) $-20 n'_{2y} = 0$

3.) $-20 n'_{2x} - 10 n'_{2z} = 0$

2.) $n'_{2y} = 0$

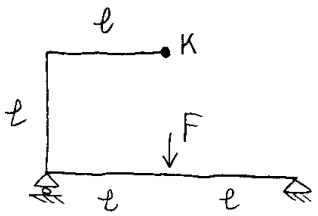
$n'_{2x} := 1$

1.) $n'_{2z} = \frac{40}{20} n'_{2x} = 2$

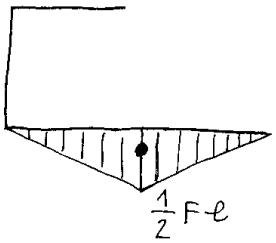
$$\underline{n}'_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \qquad |\underline{n}'_2| = \sqrt{1^2 + 2^2} = \sqrt{5}$$

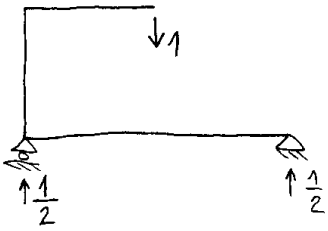
$$\underline{n}_2 = \frac{\pm \underline{n}'_2}{|\underline{n}'_2|} = \frac{\pm 1}{\sqrt{5}}\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \pm\begin{bmatrix} 0{,}4472 \\ 0 \\ 0{,}8944 \end{bmatrix}$$

(4)

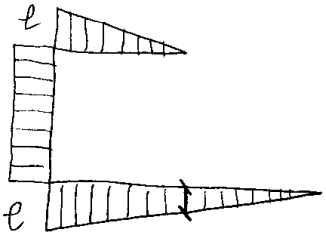


(M)

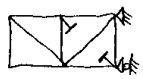




(m)

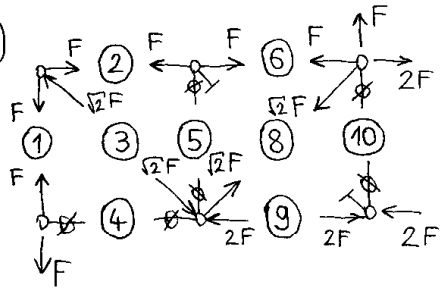


$$f_y^{(K)} = \int \frac{M m}{IE}\, ds = \frac{1}{IE}\left[\frac{1}{2}\cdot \ell \cdot \frac{1}{2} F \ell \cdot \frac{2}{3}\left(\frac{1}{2}\ell\right)\right] = \frac{1}{4}\,\frac{F\ell^3}{IE}\ (\downarrow)$$

⑤ $N_7 = X_1 \rightarrow$ 

i	A_i	E_i	l_i	N_{i0}	n_{i1}	$\frac{N_{i0} n_{i1} l_i}{A_i E_i}$	$\frac{n_{i1}^2 l_i}{A_i E_i}$	$N_i = N_{i0} + X_1 n_{i1}$
1			a	$+F$	0	0	0	$+F$ (h)
2			a	$+F$	0	0	0	$+F$ (h)
3			$\sqrt{2}\,a$	$-\sqrt{2}F$	0	0	0	$-\sqrt{2}F = -1{,}414F$ (ny)
4			a	0	0	0	0	0
5	A	E	a	0	$\frac{-1}{\sqrt{2}}$	0	$\frac{1}{2}\frac{a}{AE}$	$+0{,}3965F$ (h)
6			a	$+F$	$\frac{-1}{\sqrt{2}}$	$\frac{-\sqrt{2}}{2}\frac{Fa}{AE}$	$\frac{1}{2}\frac{a}{AE}$	$+1{,}396F$ (h)
7			$\sqrt{2}\,a$	0	$+1$	0	$\sqrt{2}\frac{a}{AE}$	$-0{,}5607F$ (ny)
8			$\sqrt{2}\,a$	$+\sqrt{2}F$	$+1$	$+2\frac{Fa}{AE}$	$\sqrt{2}\frac{a}{AE}$	$+0{,}8535F$ (h)
9			a	$-2F$	$\frac{-1}{\sqrt{2}}$	$+\sqrt{2}\frac{Fa}{AE}$	$\frac{1}{2}\frac{a}{AE}$	$-1{,}604F$ (ny)
10			a	0	$\frac{-1}{\sqrt{2}}$	0	$\frac{1}{2}\frac{a}{AE}$	$+0{,}3965F$ (h)

N_{i0}



$$\delta_{10} = \sum_{i=1}^{10} \frac{N_{i0} n_{i1} l_i}{A_i E_i} = \left(2 + \frac{\sqrt{2}}{2}\right)\frac{Fa}{AE}$$

$$\delta_{11} = \sum_{i=1}^{10} \frac{n_{i1}^2 l_i}{A_i E_i} = 2\left(1+\sqrt{2}\right)\frac{a}{AE}$$

$$N_7 = X_1 = \frac{-\delta_{10}}{\delta_{11}} = \frac{-\left(2+\frac{\sqrt{2}}{2}\right)\frac{Fa}{AE}}{2\left(1+\sqrt{2}\right)\frac{a}{AE}} =$$

$$= -0{,}5607\,F\ (ny)$$

n_{i1}