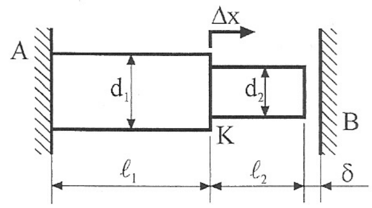
1. Feladat: *(15 pont)* Az ábrázolt alkatrész az A pontban rögzített. Szobahőmérsékleten az alkatrész és a B támasz között δ hézag van.

a) Összezáródik-e a hézag, ha az alkatrész hőmérséklete Δt értékkel növekszik?

b) Mekkorák lesznek az F_A és F_B reakcióerők?

c) Mekkora lesz a K keresztmetszet Δx elmozdulása?



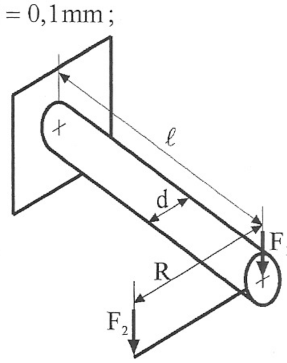
Adatok: $\ell_1 = 0{,}4\,\text{m}$; $\ell_2 = 0{,}2\,\text{m}$; $d_1 = 3\,\text{cm}$; $d_2 = 2\,\text{cm}$; $E_1 = E_2 = E = 210\,\text{GPa}$; $\delta = 0{,}1\,\text{mm}$;

$$\alpha_1 = \alpha_2 = \alpha = 1{,}2 \cdot 10^{-5}\,\frac{1}{°C};\quad \Delta t_1 = \Delta t_2 = \Delta t = +50\,°C$$

2. Feladat: *(15 pont)* A befogott kör keresztmetszetű alkatrészt az F_1 és F_2 erő terheli.

Méretezze az alkatrészt Mohr-elmélet szerint! (d=?)

Adatok: $F_1 = 50\,\text{N}$; $F_2 = 100\,\text{N}$; $\ell = 0{,}7\,\text{m}$; $R = 0{,}5\,\text{m}$; $\sigma_{meg} = 140\,\text{MPa}$

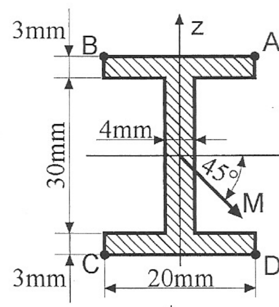


3. Feladat: *(15 pont)* Az ábrázolt keresztmetszetre az M hajlítónyomaték hat.

a) Adja meg előjelesen az A, B, C és D pontokban a normálfeszültség értékét!

b) Számítsa ki $(\beta = ?)$, és ábrázolja a semleges tengely helyzetét!

Adatok: $M = 50\,\text{Nm}$

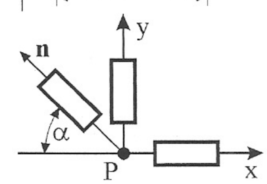


4. Feladat: *(15 pont)* A P felületi pont közelében a megadott fajlagos nyúlásokat mértük.

a) Határozza meg az **A** alakváltozási és az **F** feszültségi tenzort!

b) Határozza meg a P pontban a HMH elmélet szerinti redukált feszültséget (σ_{HMH})!

Adatok: $\varepsilon_x = -2 \cdot 10^{-4}$; $\varepsilon_y = 6 \cdot 10^{-4}$; $\varepsilon_n = 6 \cdot 10^{-4}$; $m = 3$; $E = 160\,\text{GPa}$

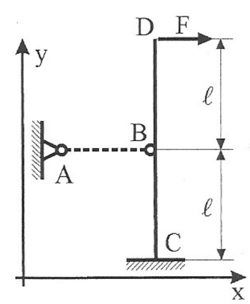


5. Feladat: *(20 pont)* A vázolt szerkezetben az A-B kötél K_0 erő hatására elszakad. (A szakadásig ideális, nyújthatatlan kötélként modellezhető.) A CD gerenda állandó keresztmetszetű.

a) Mekkora F erőnél szakad el a kötél?

b) Rajzolja meg a gerenda hajlító nyomatéki ábráját!

Adatok: $K_0 = 3600\,\text{N}$; $I = 440\,\text{cm}^4$; $E = 2 \cdot 10^5\,\text{MPa}$; $\ell = 0{,}8\,\text{m}$

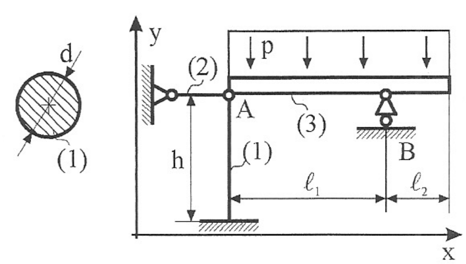


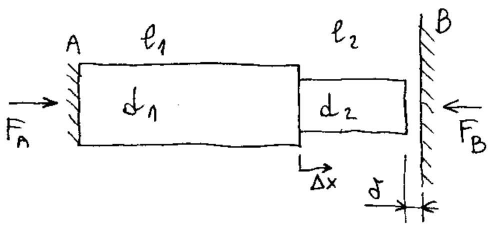
6. Feladat: *(15 pont)* A (3) merev gerendát p intenzitású megoszló erőrendszer terheli. A (2) rúd merev. Az (1) rúd tömör kör keresztmetszetű.

Méretezze az (1) rudat (d=?) kihajlásra a megfelelő biztonság figyelembe vételével!

Adatok: $\ell_1 = 5\,\text{m}$; $\ell_2 = 2\,\text{m}$; $h = 4\,\text{m}$; $p = 2\,\dfrac{\text{kN}}{\text{m}}$; $n = 3$; $\lambda_A = 100$;

$E = 200\,\text{GPa}$





$\ell_1 = 0{,}4\,m \qquad \ell_2 = 0{,}2\,m$

$d_1 = 3\,cm \qquad d_2 = 2\,cm$

$E_1 = E_2 = E = 210\,GPa$

$\alpha_1 = \alpha_2 = \alpha = 1{,}2 \cdot 10^{-5}\ \frac{1}{°C}$

$\Delta t_1 = \Delta t_2 = \Delta t = +50°C$

$\delta = 0{,}1\,mm$

$$\lambda_t = \lambda_{t_1} + \lambda_{t_2} = \alpha \ell_1 \Delta t + \alpha \ell_2 \Delta t = \alpha(\ell_1 + \ell_2)\Delta t$$

$$\lambda_t = 1{,}2 \cdot 10^{-5}(400+200)\cdot 50 = 0{,}36\,mm > \delta$$

$$F_A = F_B = F \quad ; \quad N_1 = N_2 = -F$$

$$A_1 = \frac{d_1^2 \pi}{4} = \frac{30^2 \pi}{4} = 706{,}9\,mm^2$$

$$A_2 = \frac{d^2 \pi}{4} = \frac{20^2 \pi}{4} = 314{,}2\,mm^2$$

$$\lambda_t + \lambda = \delta$$

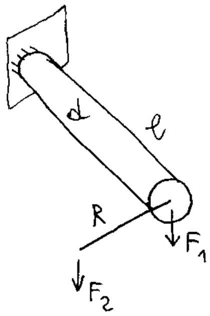
$$\lambda_t + \frac{(-F)\ell_1}{A_1 E} + \frac{(-F)\ell_2}{A_2 E} = \delta$$

$$F = \frac{E(\lambda_t - \delta)}{\frac{\ell_1}{A_1} + \frac{\ell_2}{A_2}} = \frac{210 \cdot 10^3 (0{,}36 - 0{,}1)}{\frac{400}{706{,}9} + \frac{200}{314{,}2}} = 45410\,N$$

$$\boxed{F_A = F_B = F = 45410\,N}$$

$$\Delta x = \lambda_{t_1} + \lambda_1 = \alpha \ell_1 \Delta t + \frac{(-F)\ell_1}{A_1 E} = 1{,}2 \cdot 10^{-5} \cdot 400 \cdot 50 - \frac{45410 \cdot 400}{706{,}9 \cdot 210 \cdot 10^3}$$

$$\boxed{\Delta x = 0{,}24 - 0{,}1224 = +0{,}1176\,mm}$$



$\sigma_{meg} = 140$ MPa

$R = 0,5$ m

$\ell = 0,7$ m

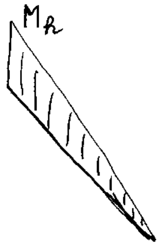
$F_1 = 50$ N

$F_2 = 100$ N

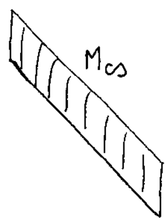
MOHR - elmélet

$d = ?$

M_h



M_{cs}



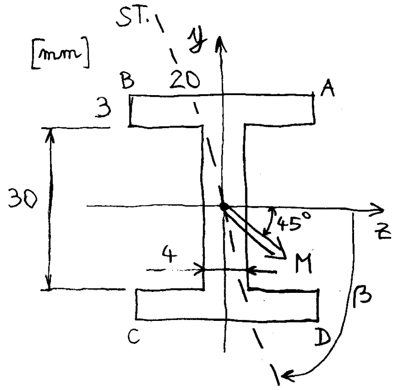
$$M_h = (F_1 + F_2)\,\ell = (50+100)\cdot 0,7 = 105 \text{ Nm}$$

$$M_{cs} = F_2 \cdot R = 100 \cdot 0,5 = 50 \text{ Nm}$$

$$M_{red,MOHR} = \sqrt{M_h^2 + M_{cs}^2} = \sqrt{105^2 + 50^2} = 116,3 \text{ Nm}$$

$$\sigma_{meg} \geqq \sigma_{red,MOHR} = \frac{M_{red,MOHR}}{I}\,\frac{d}{2} = \frac{M_{red,MOHR}}{\frac{d^4 \pi}{64}}\,\frac{d}{2} = \frac{32\, M_{red,MOHR}}{d^3\, \pi}$$

$$\boxed{d \geq \sqrt[3]{\frac{32\, M_{red,MOHR}}{\sigma_{meg}\, \pi}} = \sqrt{\frac{32 \cdot 116,3 \cdot 10^3}{140\, \pi}} = \boxed{20,38 \text{ mm}}}$$



$M = 50\ \mathrm{Nm}$

1p

$\alpha = -45^\circ$

$$I_z = \frac{36^3 \cdot 20}{12} - \frac{30^3 \cdot 16}{12} = 77760 - 36000 = 41\,760\ \mathrm{mm}^4$$ 2p

$$I_y = \frac{4^3 \cdot 30}{12} + 2 \cdot \frac{20^3 \cdot 3}{12} = 160 + 2 \cdot 2000 = 4160\ \mathrm{mm}^4$$ 2p

$$\mathrm{tg}\,\beta = \frac{I_z}{I_y}\,\mathrm{tg}\,\alpha = \frac{41\,760}{4160}\,\mathrm{tg}(-45^\circ) = -10{,}04 \rightarrow \boxed{\beta = -84{,}31^\circ}$$ 2p

$$M_z = M\cos\alpha = 50 \cdot \cos(-45^\circ) = +35{,}36\ \mathrm{Nm}$$
$$M_y = M\sin\alpha = 50\sin(-45^\circ) = -35{,}36\ \mathrm{Nm}$$ 2p

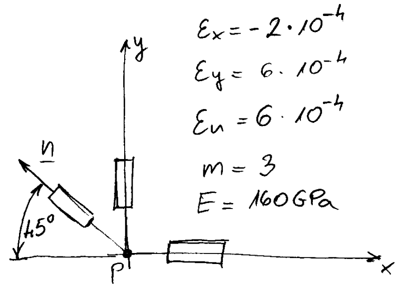
$$\sigma_A = \frac{M_z}{I_z}y_A - \frac{M_y}{I_y}x_A = \frac{+35{,}36 \cdot 10^3}{41760}(+18) - \frac{-35{,}36 \cdot 10^3}{4160}(+10)$$

$$\boxed{\sigma_A = 0{,}8467 \cdot 18 + 8{,}500 \cdot 10 = 15{,}24 + 85 = 100{,}2\ \mathrm{MPa}}$$ 2p

$$\boxed{\sigma_B = \frac{M_z}{I_z}y_B - \frac{M_y}{I_y}x_B = 0{,}8467(+18) + 8{,}5(-10) = 15{,}24 - 85 = -69{,}76\ \mathrm{MPa}}$$ 2p

$$\boxed{\sigma_C = -\sigma_A = -100{,}2\ \mathrm{MPa}}$$ 1p

$$\boxed{\sigma_D = -\sigma_B = +69{,}76\ \mathrm{MPa}}$$ 1p



$\varepsilon_x = -2\cdot10^{-4}$

$\varepsilon_y = 6\cdot10^{-4}$

$\varepsilon_n = 6\cdot10^{-4}$

$m = 3$

$E = 160\,GPa$

$$\underline{\underline{A}} = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{yx} & 0 \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix} \qquad \underline{n} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$$

$$\underline{\underline{F}} = \begin{bmatrix} \sigma_x & \tau_{yx} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\sigma_z = 0 = 2G\left(\varepsilon_z + \frac{\varepsilon_x+\varepsilon_y+\varepsilon_z}{m-2}\right) \Rightarrow \quad \varepsilon_z = -\frac{\varepsilon_x+\varepsilon_y}{m-1} = -2\cdot10^{-4}$$

$$\varepsilon_n = \underline{n}^T\underline{\underline{A}}\underline{n} = \underline{n}^T\cdot\begin{bmatrix} -\frac{\varepsilon_x}{\sqrt{2}} + \frac{\gamma_{yx}}{2\sqrt{2}} \\ -\frac{\gamma_{xy}}{2\sqrt{2}} + \frac{\varepsilon_y}{\sqrt{2}} \\ 0 \end{bmatrix} = \left(\frac{\varepsilon_x}{2} - \frac{\gamma_{yx}}{4}\right) + \left(-\frac{\gamma_{xy}}{4} + \frac{\varepsilon_y}{2}\right) =$$

$$= \frac{\varepsilon_x}{2} - \frac{\gamma_{xy}}{2} + \frac{\varepsilon_y}{2} = 2\cdot10^{-4} - \frac{\gamma_{xy}}{2} = 6\cdot10^{-4} = \varepsilon_n$$

$$\Rightarrow \quad \frac{\gamma_{xy}}{2} = -4\cdot10^{-4}$$

$$\underline{\underline{A}} = \begin{bmatrix} -2 & -4 & 0 \\ -4 & 6 & 0 \\ 0 & 0 & -2 \end{bmatrix}\cdot10^{-4}$$

$$\sigma_x = 2G\left(\varepsilon_x + \frac{\varepsilon_x+\varepsilon_y+\varepsilon_z}{m-2}\right) = 120\cdot10^9\left(-2 + \frac{-2+6-2}{1}\right)\cdot10^{-4} = 0\,MPa$$

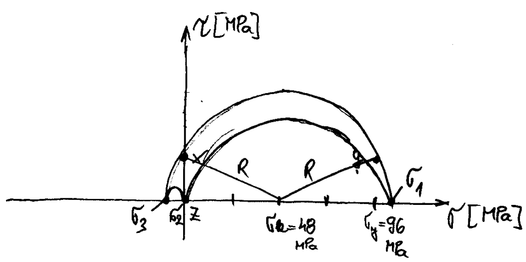
$$G = \frac{E}{2(1+\frac{1}{m})} = \frac{160}{2(1+\frac{1}{3})} = 60\,GPa$$

$$\underline{\underline{F}} = \begin{array}{c} \begin{bmatrix} 0 & -48 & 0 \\ -48 & 96 & 0 \\ 0 & 0 & 0 \end{bmatrix} MPa \\ \begin{matrix} x & \quad y & \quad z \end{matrix} \end{array}$$

$$\sigma_y = 2G\left(\varepsilon_y + \frac{\varepsilon_x+\varepsilon_y+\varepsilon_z}{m-2}\right) = 120\cdot10^9\left(6 + \frac{-2+6-2}{1}\right)\cdot10^{-4} =$$

$$= 96\cdot10^6\,Pa = 96\,MPa$$

$$\tau_{xy} = G\cdot\gamma_{xy} = 60\cdot10^9\cdot(-8\cdot10^{-4}) = -24\cdot10^6\,Pa = -48\,MPa$$



z : főirány

$$\sigma_k = \frac{\sigma_x+\sigma_y}{2} = \frac{0+96}{2} = 48\,MPa$$

$$R = \sqrt{\left(\frac{\sigma_y-\sigma_x}{2}\right)^2 + \tau_{xy}^2} = \sqrt{48^2+24^2} = 53{,}67\,MPa$$

$$\sigma_{red}^{HMH} = \sqrt{F_I^2 - 3F_{II}} = \sqrt{96^2+3\cdot48^2} =$$

$$= 127\,MPa$$

$$F_{II} = 0 + 0 - (-48^2) = -48^2 = -2304$$

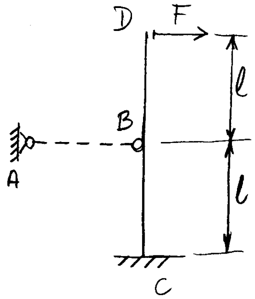
$$\sigma_1 = \sigma_k + R = 48 + 53{,}67 = 101{,}67\,MPa$$

$$\sigma_2 = \sigma_z = 0\,MPa$$

$$\sigma_3 = \sigma_k - R = 48 - 53{,}67 = -5{,}67\,MPa$$

$$\sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]} = 104{,}62\,MPa$$

5.



$K_0 = 3600\,N \quad I = 440\,cm^4 \quad E = 2\cdot 10^5\,MPa \quad l = 0{,}8\,m$

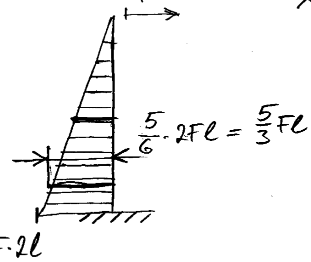
$$\delta_{10} = \int_{(2l)} \frac{M_0 \cdot m_1}{IE}\,dx = 0 + \frac{1}{IE}\cdot\frac{l^2}{2}\cdot\frac{-5Fl}{3} = \frac{-5Fl^3}{6IE}$$

$$\delta_{11} = \int_{(2l)} \frac{m_1 \cdot m_1}{IE}\,dx = \frac{1}{IE}\cdot\frac{l^2}{2}\cdot\frac{2}{3}l = \frac{l^3}{3IE}$$

Törzstartó

$$X_1 = -\frac{\delta_{10}}{\delta_{11}} = \frac{\frac{5Fl^3}{6IE}}{\frac{l^3}{3IE}} = \frac{\frac{5}{6}F}{\frac{1}{3}} = \frac{5}{2}F$$

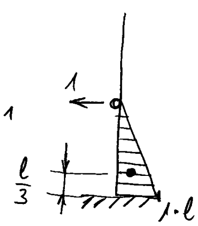
M_0

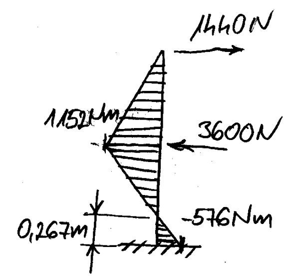


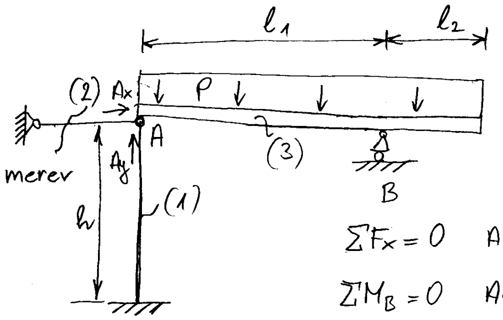
$$K_0 = X_1 = \frac{5}{2}F \implies F = \frac{2}{5}K_0 = \frac{2}{5}\cdot 3600 =$$

$$= 1440\,N$$

m_1







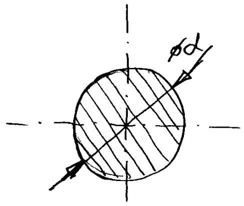
$l_1 = 5m$ $\quad p = 2\frac{kN}{m}$ $\quad E = 200GPa$

$l_2 = 2m$ $\quad n = 3$ $\quad d = ?$

$h = 4m$ $\quad \lambda_A = 100$

$\sum F_x = 0 \quad A_x = 0$ (1p)

$\sum M_B = 0 \quad A_y \cdot l_1 - p(l_1+l_2)\cdot\left(\frac{l_1+l_2}{2} - l_2\right) = 0$



$$A_y = \frac{p(l_1+l_2)(l_1-l_2)}{2\cdot l_1} = \frac{2000\cdot(5+2)(5-2)}{2\cdot 5} = 4200\,N$$ (2p)

$F_{krit} = n\cdot A_y = 3\cdot 4200 = 12600\,N$ (1p)

$$F_{krit} = \frac{\pi^2 \cdot E \cdot I}{l_0^2} = \frac{\pi^2 \cdot 2\cdot 10^{11} \cdot \frac{d^4\pi}{64}}{\left(\frac{4}{\sqrt{2}}\right)^2} \Rightarrow d = \sqrt[4]{\frac{F_{krit}\cdot 32\cdot h^2}{\pi^3\cdot E}} =$$

$l_0 = \frac{h}{\sqrt{2}} \approx 0{,}7h$ (3p)

$$= \sqrt[4]{\frac{12600\cdot 32\cdot 4^2}{\pi^3\cdot 2\cdot 10^{11}}} = 0{,}0319\,m = 31{,}9mm$$ (3p)

$\underline{\underline{d = 31{,}9\,mm}}$

$\underline{Ell.}$: $I = \frac{d^4\pi}{64} = 5{,}083\cdot 10^{-8}\,m^4$ $\qquad i = \sqrt{\frac{I}{A}} = 0{,}00797\,m$

$A = \frac{d^2\pi}{4} = 7{,}992\cdot 10^{-4}\,m^2$ $\qquad \lambda = \frac{l_0}{i} = \frac{4}{\sqrt{2}\cdot 0{,}00797} = 354{,}7$

(5p)

$\lambda > \lambda_A = 100$ ✓