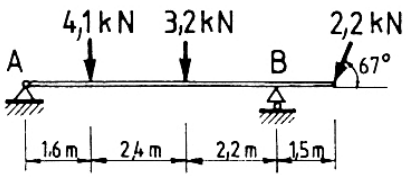
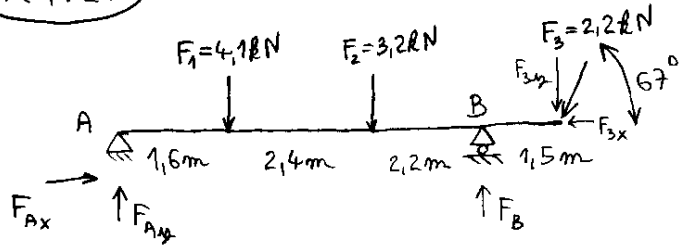
F.4.27.b.: A 2010.03.04.-ei gyakorlat 5. példája.



F.4.27. Határozzuk meg a reakcióerőket *a)* szerkesztéssel, *b)* számítással!

F.4.27.



$$\Sigma F_x = 0 = F_{Ax} - F_{3x} \rightarrow \boxed{F_{Ax} = F_{3x} = 2{,}2 \cos 67^\circ = 0{,}860\ \text{kN}\ (\rightarrow)}$$

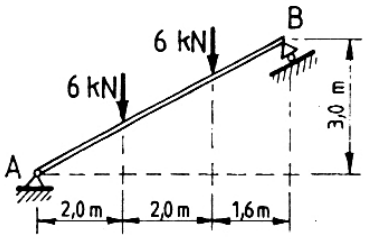
$$\Sigma M_A = 0 = -F_1 \cdot 1{,}6 - F_2 \cdot 4 + F_B \cdot 6{,}2 - F_{3y} \cdot 7{,}7$$

$$\boxed{F_B = \frac{1{,}6F_1 + 4F_2 + 7{,}7F_{3y}}{6{,}2} = \frac{1{,}6 \cdot 4{,}1 + 4 \cdot 3{,}2 + 7{,}7 \cdot 2{,}2 \sin 67^\circ}{6{,}2} = 5{,}578\ \text{kN}\ (\uparrow)}$$

$$\Sigma F_y = 0 = F_{Ay} - F_1 - F_2 + F_B - F_{3y}$$

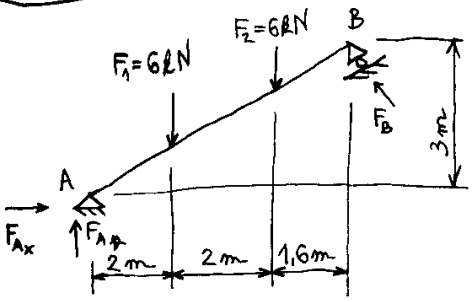
$$\boxed{F_{Ay} = F_1 + F_2 - F_B + F_{3y} = 4{,}1 + 3{,}2 - 5{,}578 + 2{,}2 \sin 67^\circ = 3{,}747\ \text{kN}\ (\uparrow)}$$

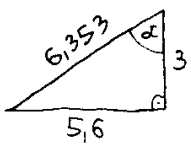
F.4.28.: A 2010.03.04.-ei gyakorlat 6. példája.

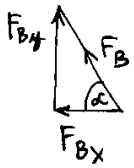


F.4.28. Határozzuk meg a reakcióerőket *a)* szerkesztéssel, *b)* számítással!

F.4.28.







$$\operatorname{tg}\alpha = \frac{5,6}{3} \rightarrow \alpha = 61,82^\circ$$

$$\sum M_A = 0 = -F_1 \cdot 2 - F_2 \cdot 4 + F_B \cdot 6,353$$

$$F_B = \frac{2F_1 + 2F_2}{6,353} = \frac{2 \cdot 6 + 4 \cdot 6}{6,353} = 5,667\ kN\ (\nwarrow)$$

$$\sum F_x = 0 = F_{Ax} - F_{Bx}$$

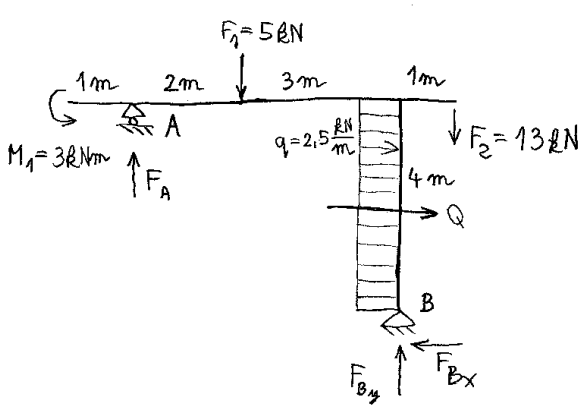
$$F_{Ax} = F_{Bx} = 5,667 \cos 61,82^\circ = 2,676\ kN\ (\rightarrow)$$

$$\sum F_y = 0 = F_{Ay} - F_1 - F_2 + F_{By}$$

$$F_{Ay} = F_1 + F_2 - F_{By} = 6 + 6 - 5,667 \sin 61,82^\circ = 7,005\ kN\ (\uparrow)$$

DS-1.: A 2010.03.05.-ei gyakorlat 1. példája.

Számítsuk ki a reakcióerőket!



$Q = q \cdot 4 = 2{,}5 \cdot 4 = 10\ kN$

$$\sum F_x = 0 = Q - F_{Bx} \rightarrow \boxed{F_{Bx} = Q = 10\ kN\ (\leftarrow)}$$

$$\sum M_B = 0 = M_1 - F_A \cdot 5 + F_1 \cdot 3 - F_2 \cdot 1 - Q \cdot 2$$

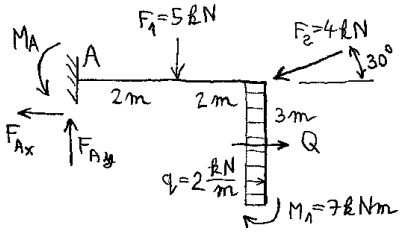
$$\boxed{F_A = \frac{M_1 + 3F_1 - F_2 - 2Q}{5} = \frac{3 + 3 \cdot 5 - 13 - 2 \cdot 10}{5} = -3\ kN\ (\downarrow)}$$

$$\sum F_y = 0 = F_A - F_1 - F_2 + F_{By}$$

$$\boxed{F_{By} = F_1 + F_2 - F_A = 5 + 13 - (-3) = 21\ kN\ (\uparrow)}$$

DS-2.: A 2010.03.05.-ei gyakorlat 2. példája.

Számítsuk ki a reakcióerőket!



$Q = q \cdot 3 = 2 \cdot 3 = 6\ kN$

$\sum F_x = 0 = -F_{Ax} - F_{2x} + Q \rightarrow \boxed{F_{Ax} = Q - F_{2x} = 6 - 4\cos 30^\circ = +2{,}536\ kN\ (\leftarrow)}$

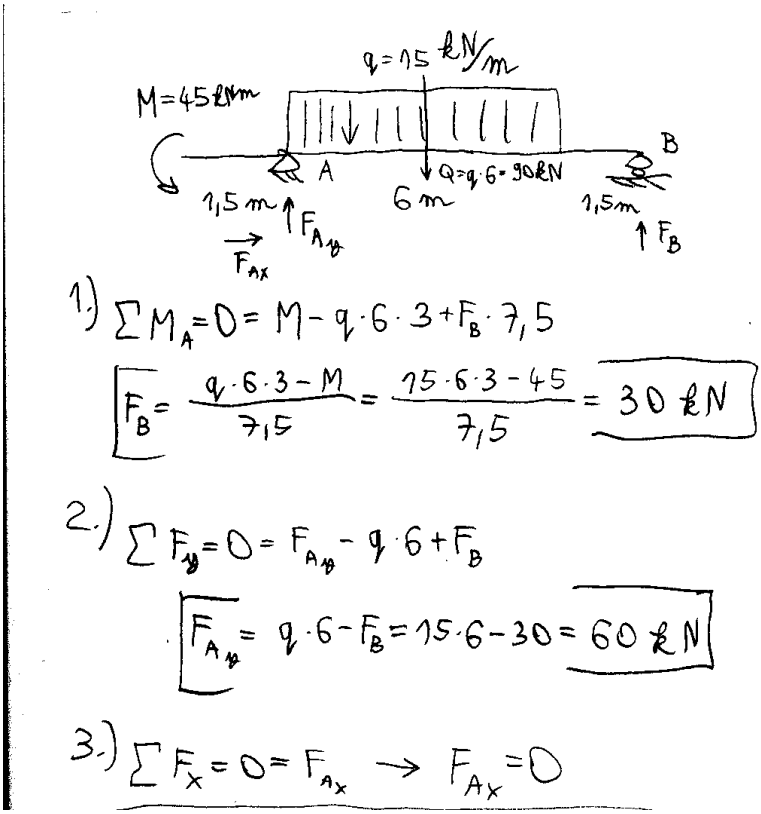
$\sum F_y = 0 = F_{Ay} - F_1 - F_{2y} \rightarrow \boxed{F_{Ay} = F_1 + F_{2y} = 5 + 4\sin 30^\circ = 7\ kN\ (\uparrow)}$

$\sum M_A = 0 = M_A - F_1 \cdot 2 - F_{2y} \cdot 4 + Q \cdot 1{,}5 - M_1$

$\boxed{M_A = 2F_1 + 4F_{2y} - 1{,}5Q + M_1 = 2 \cdot 5 + 4 \cdot 4\sin 30^\circ - 1{,}5 \cdot 6 + 7 = 16\ kNm\ \circlearrowleft}$

További feladatok:

DS-3.: Számítsuk ki a reakcióerőket!



1.) $\sum M_A = 0 = M - q \cdot 6 \cdot 3 + F_B \cdot 7{,}5$

$$F_B = \frac{q \cdot 6 \cdot 3 - M}{7{,}5} = \frac{15 \cdot 6 \cdot 3 - 45}{7{,}5} = 30\ kN$$

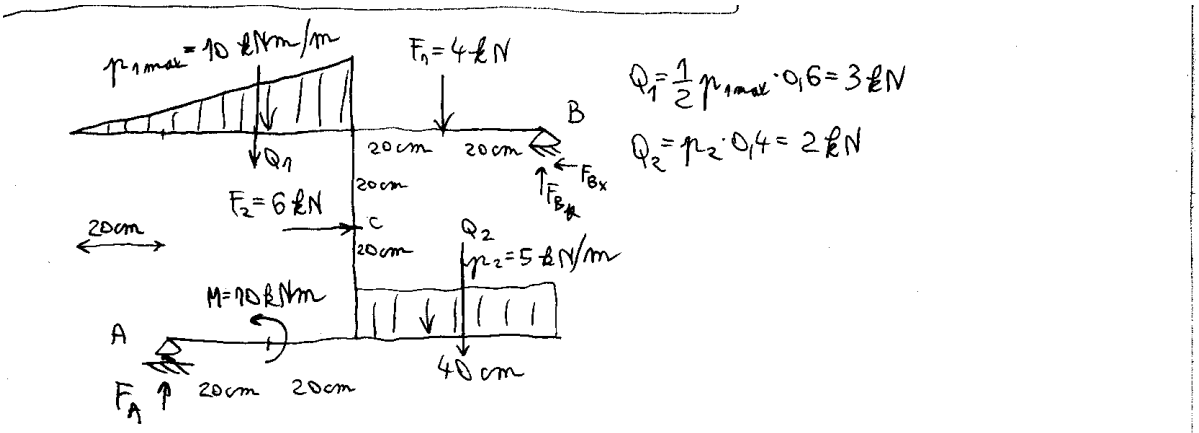
2.) $\sum F_y = 0 = F_{Ay} - q \cdot 6 + F_B$

$$F_{Ay} = q \cdot 6 - F_B = 15 \cdot 6 - 30 = 60\ kN$$

3.) $\sum F_x = 0 = F_{Ax} \rightarrow F_{Ax} = 0$

DS-4.: Számítsuk ki a reakcióerőket!

Megjegyzés: Ezen van egy lineárisan megoszló terhelés is.



$Q_1 = \frac{1}{2} p_{1max} \cdot 0{,}6 = 3\ kN$

$Q_2 = p_2 \cdot 0{,}4 = 2\ kN$

1.) $\sum M_B = 0 = -Q_1 \cdot 0{,}6 + F_1 \cdot 0{,}2 + F_2 \cdot 0{,}2 - F_A \cdot 0{,}8 + M + Q_2 \cdot 0{,}2$

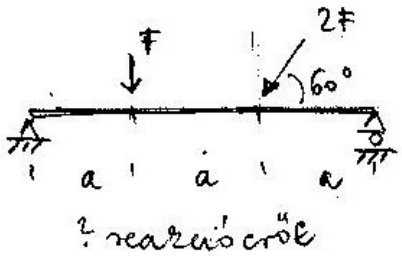
$$F_A = \frac{1}{0{,}8}\left[M + 0{,}6\,Q_1 + 0{,}2\left(F_1 + F_2 + Q_2\right)\right] = \frac{1}{0{,}8}\left[10 + 0{,}6 \cdot 3 + 0{,}2\left(4 + 6 + 2\right)\right] = 17{,}75\ kN\ (\uparrow)$$

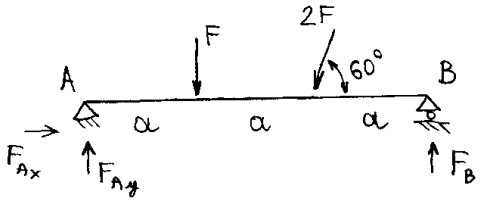
2.) $\sum F_y = 0 = -Q_1 - F_1 + F_A - Q_2 + F_{By}$

$$F_{By} = Q_1 + F_1 - F_A + Q_2 = 3 + 4 - 17{,}75 + 2 = -8{,}75\ kN\ (\downarrow)$$

3.) $\sum F_x = 0 = F_2 - F_{Bx} \rightarrow F_{Bx} = F_2 = 6\ kN\ (\leftarrow)$

Sz-1: Határozzuk meg a reakcióerőket!





$$\sum F_x = 0 = F_{Ax} - 2F\cos 60^\circ$$

$$\boxed{F_{Ax} = 2F\cos 60^\circ = 2F \cdot \frac{1}{2} = F\ (\rightarrow)}$$

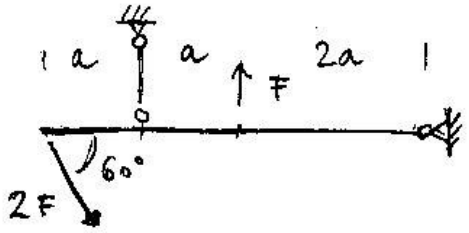
$$\sum M_A = 0 = -F \cdot a - 2F \sin 60^\circ \cdot 2a + F_B \cdot 3a$$

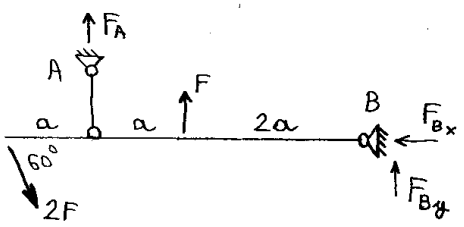
$$\boxed{F_B = \frac{F + 4F\sin 60^\circ}{3} = 1{,}488\,F\ (\uparrow)}$$

$$\sum F_y = 0 = F_{Ay} - F - 2F\sin 60^\circ + F_B$$

$$\boxed{F_{Ay} = F + 2F\sin 60^\circ - F_B = F + 2F\sin 60^\circ - (1{,}488\,F) = 1{,}244F\ (\uparrow)}$$

Sz-2: Határozzuk meg a reakcióerőket!





$$\sum F_x = 2F\cos 60^\circ - F_{Bx}$$

$$\boxed{F_{Bx} = 2F\cos 60^\circ = 2F \cdot \frac{1}{2} = F\ (\leftarrow)}$$

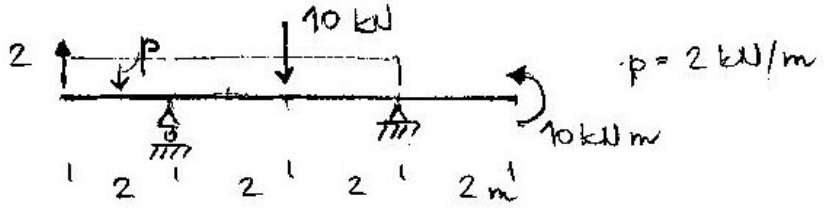
$$\sum M_B = 0 = 2F\sin 60^\circ \cdot 4a - F_A \cdot 3a - F \cdot 2a$$

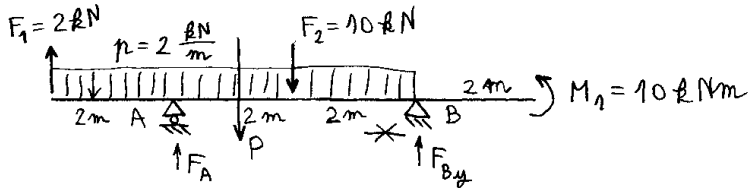
$$\boxed{F_A = \frac{8F\sin 60^\circ - 2F}{3} = 1{,}642\ F\ (\uparrow)}$$

$$\sum F_y = 0 = -2F\sin 60^\circ + F_A + F + F_{By}$$

$$\boxed{F_{By} = 2F\sin 60^\circ - F_A - F = 2F\sin 60^\circ - (1{,}642F) - F = -0{,}9099\ F\ (\downarrow)}$$

Sz-3: Határozzuk meg a reakcióerőket!





$\sum F_x = 0 \rightarrow \boxed{F_{Bx} = 0}$

$P = p\,\ell = 2 \cdot 6 = 12\ kN$

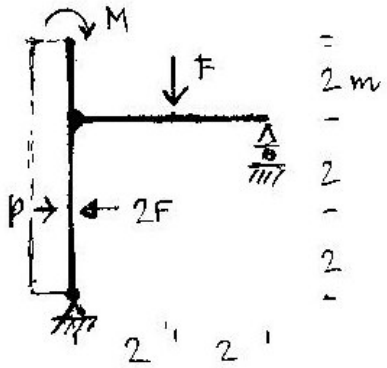
$\sum M_B = -F_1 \cdot 6 - F_A \cdot 4 + P \cdot 3 + F_2 \cdot 2 + M_1$

$$\boxed{F_A = \frac{-6F_1 + 3P + 2F_2 + M_1}{4} = \frac{-6 \cdot 2 + 3 \cdot 12 + 2 \cdot 10 + 10}{4} = 13{,}5\ kN\ (\uparrow)}$$

$\sum F_y = 0 = F_1 + F_A - P - F_2 + F_{By}$

$$\boxed{F_{By} = -F_1 - F_A + P + F_2 = -2 - 13{,}5 + 12 + 10 = 6{,}5\ kN\ (\uparrow)}$$

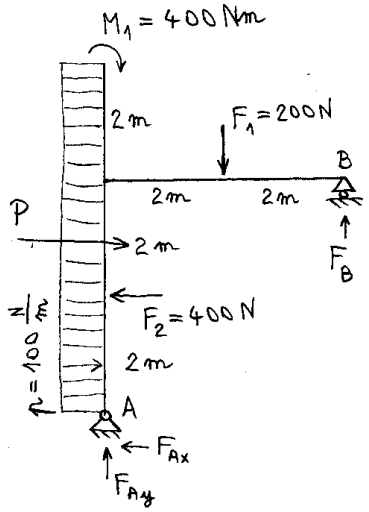
Sz-4: Határozzuk meg a reakcióerőket!



$p = 100\ N/m$

$M = 400\ Nm$

$F = 200\ N$



$P = p\,\ell = 100 \cdot 6 = 600\ N$

$\sum F_x = 0 = P - F_2 - F_{Ax}$

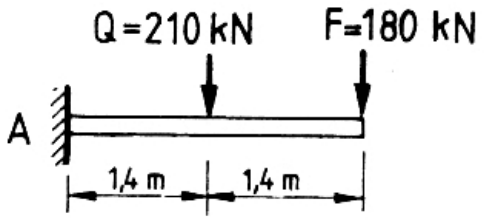
$$\boxed{F_{Ax} = P - F_2 = 600 - 400 = 200\ N}$$

$\sum M_A = 0 = F_2 \cdot 2 - P \cdot 3 - M_1 - F_1 \cdot 2 + F_B \cdot 4$

$$\boxed{F_B = \frac{-2F_2 + 3P + M_1 + 2F_1}{4} = \frac{-2 \cdot 400 + 3 \cdot 600 + 400 + 2 \cdot 200}{4} = 450\ N\ (\uparrow)}$$

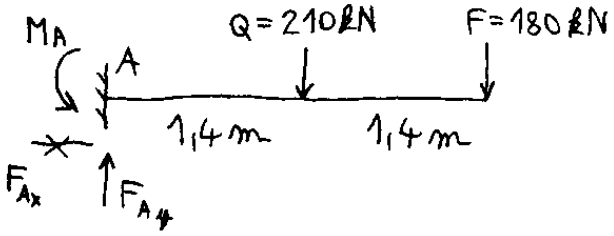
$\sum F_y = 0 = F_{Ay} - F_1 + F_B$

$$\boxed{F_{Ay} = F_1 - F_B = 200 - 450 = -250\ N\ (\downarrow)}$$



F.4.13. Számítsuk ki a reakciókat!

F.4.13.



$\sum F_x = 0 \rightarrow \boxed{F_{Ax} = 0}$

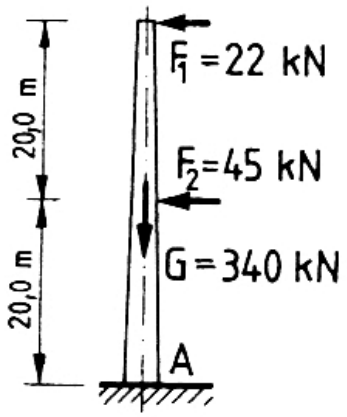
$\sum F_y = 0 = F_{Ay} - Q - F$

$\boxed{F_{Ay} = Q + F = 210 + 180 = 390\ kN\ (\uparrow)}$

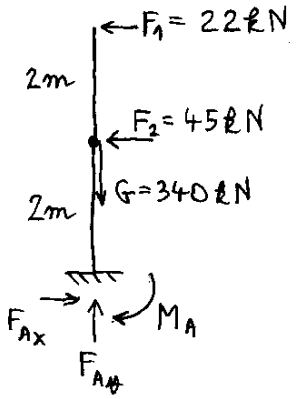
$\sum M_A = 0 = M_A - Q \cdot 1{,}4 - F \cdot 2{,}8$

$\boxed{M_A = 1{,}4\,Q + 2{,}8\,F = 1{,}4 \cdot 210 + 2{,}8 \cdot 180 = 798\ kNm\ \text{G}}$

F.4.14. Számítsuk ki a reakciókat!



F.4.14.



$\sum F_x = 0 = F_{Ax} - F_1 - F_2$

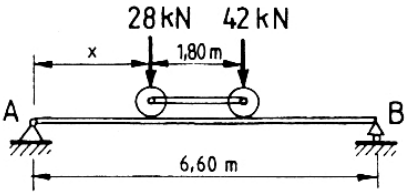
$F_{Ax} = F_1 + F_2 = 22 + 45 = 67\ kN\ (\rightarrow)$

$\sum F_y = 0 = F_{Ay} - G$

$F_{Ay} = G = 340\ kN\ (\uparrow)$

$\sum M_A = 0 = -M_A + F_1 \cdot 4 + F_2 \cdot 2$

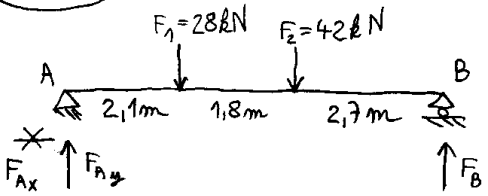
$M_A = 4F_1 + 2F_2 = 4 \cdot 22 + 2 \cdot 45 = 178\ kNm\ \circlearrowright$



F.4.23. Határozzuk meg *a)* szerkesztéssel, *b)* számítással a reakcióerőket, ha $x = 2{,}1$ m!

F.4.24. Hova helyezzük el (x = ?) a két összekapcsolt keréksúlyt, hogy a két reakcióerő egyenlő nagy legyen?

F. 4. 23.



$\Sigma F_x = 0 \rightarrow \boxed{F_{Ax} = 0}$

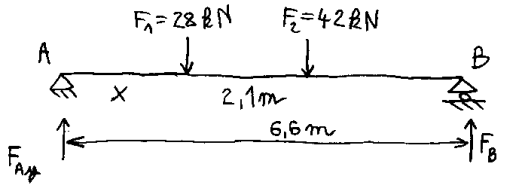
$\Sigma M_A = 0 = -F_1 \cdot 2{,}1 - F_2 \cdot 3{,}9 + F_B \cdot 6{,}6$

$$\boxed{F_B = \frac{F_1 \cdot 2{,}1 + F_2 \cdot 3{,}9}{6{,}6} = \frac{28 \cdot 2{,}1 + 42 \cdot 3{,}9}{6{,}6} = 33{,}73 \text{ kN } (\uparrow)}$$

$\Sigma F_y = 0 = F_{Ay} - F_1 - F_2 + F_B$

$$\boxed{F_{Ay} = F_1 + F_2 - F_B = 28 + 42 - 33{,}73 = 36{,}27 \text{ kN } (\uparrow)}$$

F. 4. 24.



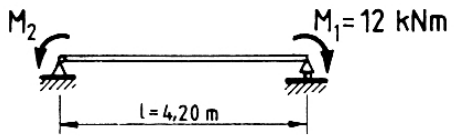
1.) $\Sigma M_A = 0 = -F_1 x - F_2 (x + 2{,}1) + F_B \cdot 6{,}6$

2.) $\Sigma F_y = 0 = F_{Ay} - F_1 - F_2 + F_B$

3.) $F_{Ay} = F_B$

3→2.) $$F_B = \frac{F_1 + F_2}{2} = \frac{28 + 42}{2} = 35 \text{ kN}$$

1.) $$\boxed{x = \frac{6{,}6\,F_B - 2{,}1\,F_2}{F_1 + F_2} = \frac{6{,}6 \cdot 35 - 2{,}1 \cdot 42}{28 + 42} = 2{,}04 \text{ m}}$$

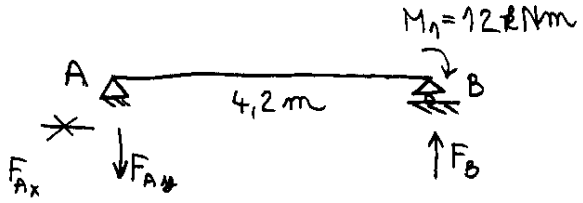


F.4.25-26. ábra

F.4.25. Számítsuk ki a reakcióerőket az $M_1 = 12$ kNm nyomatékú erőpárból ($M_2 = 0$)!

F.4.26. Számítsuk ki a reakcióerőket, ha az $M_1 =$ $= 12$ kNm nyomatékú erőpáron kívül $M_2 = 8$ kNm is működik!

F. 4. 25.

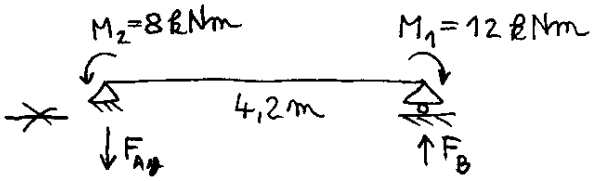


$\sum F_x = 0 \rightarrow \boxed{F_{Ax} = 0}$

$\sum M_A = 0 = -M_1 + F_B \cdot 4{,}2 \rightarrow \boxed{F_B = \frac{M_1}{4{,}2} = \frac{12}{4{,}2} = 2{,}86 \text{ kN } (\uparrow)}$

$\sum F_y = 0 = -F_{Ay} + F_B \rightarrow \boxed{F_{Ay} = F_B = 2{,}86 \text{ kN } (\downarrow)}$

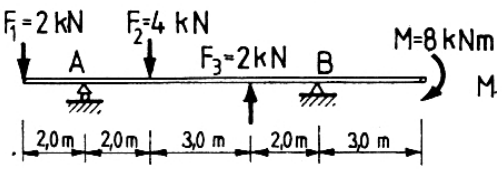
F. 4. 26.



$\sum F_x = 0 \rightarrow \boxed{F_{Ax} = 0}$

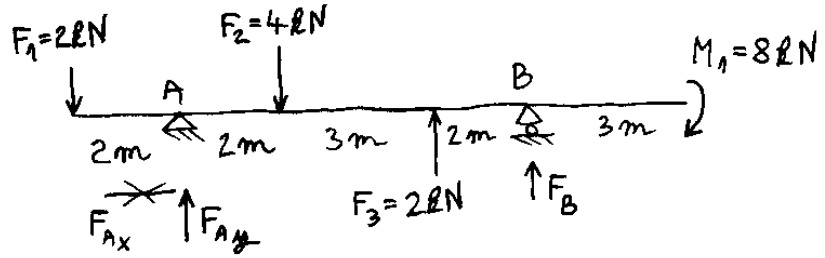
$\sum M_A = 0 = M_2 - M_1 + F_B \cdot 4{,}2 \rightarrow \boxed{F_B = \frac{M_1 - M_2}{4{,}2} = \frac{12 - 8}{4{,}2} = 0{,}95 \text{ kN } (\uparrow)}$

$\sum F_y = 0 = -F_{Ay} + F_B \rightarrow \boxed{F_{Ay} = F_B = 0{,}95 \text{ kN } (\downarrow)}$



F.4.29. Határozzuk meg a reakcióerőket *a)* szerkesztéssel, *b)* számítással!

F. 4. 29.



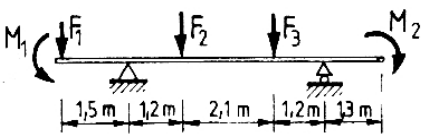
$\sum F_x = 0 \rightarrow \boxed{F_{Ax} = 0}$

$\sum M_A = 0 = +F_1 \cdot 2 - F_2 \cdot 4 + F_3 \cdot 5 + F_B \cdot 7 - M_1$

$$\boxed{F_B = \frac{-2F_1 + 4F_2 - 5F_3 + M_1}{7} = \frac{-2 \cdot 2 + 4 \cdot 4 - 5 \cdot 2 + 8}{7} = 1{,}429 \text{ kN } (\uparrow)}$$

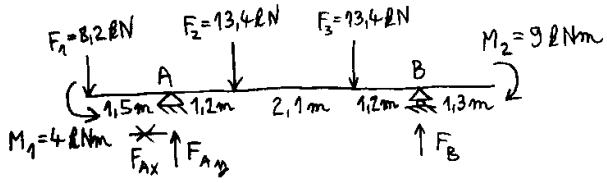
$\sum F_y = 0 = -F_1 + F_{Ay} - F_2 + F_3 + F_B$

$$\boxed{F_{Ay} = F_1 + F_2 - F_3 - F_B = 2 + 4 - 2 - 1{,}429 = 2{,}571 \text{ kN } (\uparrow)}$$



F.4.30. Határozzuk meg a reakcióerőket *a)* szerkesztéssel, *b)* számítással! $F_1 = 8{,}2$ kN, $F_2 = F_3 = 13{,}4$ kN, $M_1 = 4$ kNm, $M_2 = 9$ kNm .

F.4.30.



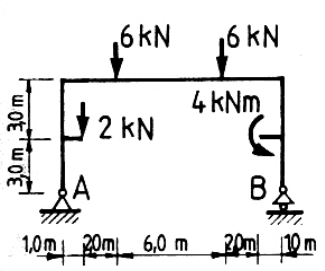
$\sum F_x = 0 \rightarrow \boxed{F_{Ax} = 0}$

$\sum M_A = 0 = M_1 + F_1 \cdot 1{,}5 - F_2 \cdot 1{,}2 - F_3 \cdot 3{,}3 + F_B \cdot 4{,}5 - M_2$

$$\boxed{F_B} = \frac{-M_1 - 1{,}5F_1 + 1{,}2F_2 + 3{,}3F_3 + M_2}{4{,}5} = \frac{-4 - 1{,}5 \cdot 8{,}2 + 1{,}2 \cdot 13{,}4 + 3{,}3 \cdot 13{,}4 + 9}{4{,}5} = \underline{11{,}178 \text{ kN } (\uparrow)}$$

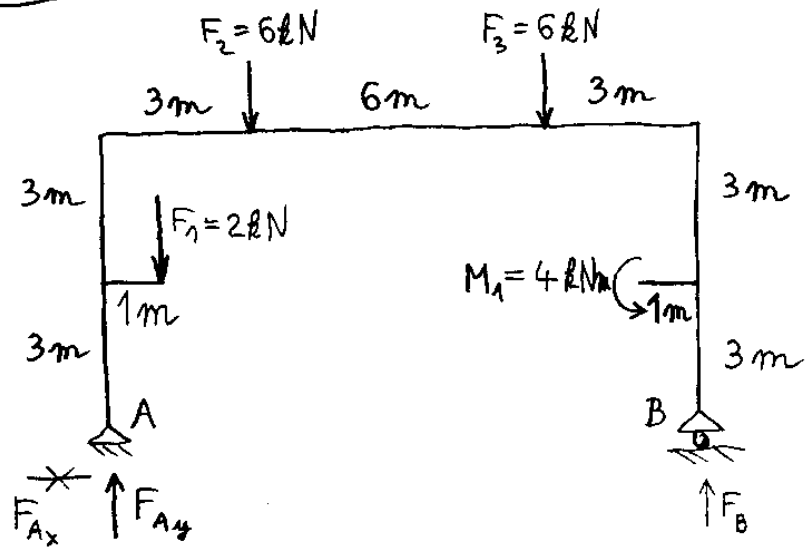
$\sum F_y = 0 = -F_1 + F_{Ay} - F_2 - F_3 + F_B$

$$\boxed{F_{Ay}} = F_1 + F_2 + F_3 - F_B = 8{,}2 + 13{,}4 + 13{,}4 - 11{,}178 = \underline{23{,}22 \text{ kN } (\uparrow)}$$



F.4.31. Számítsuk ki a reakcióerőket! Bontsuk két csoportra a terheket, szimmetrikusra és nem szimmetrikusra, és egymásra halmozással határozzuk meg a reakcióerőket!

F.4.31.



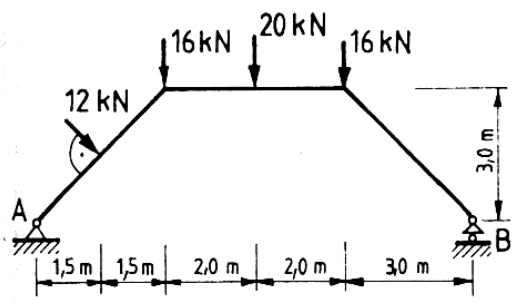
$\sum F_x = 0 \rightarrow \boxed{F_{Ax} = 0}$

$\sum M_A = 0 = -F_1 \cdot 1 - F_2 \cdot 3 - F_3 \cdot 9 + M_1 + F_B \cdot 12$

$$\boxed{F_B = \frac{F_1 + 3F_2 + 9F_3 - M_1}{12} = \frac{2 + 3 \cdot 6 + 9 \cdot 6 - 4}{12} = 5{,}833 \text{ kN } (\uparrow)}$$

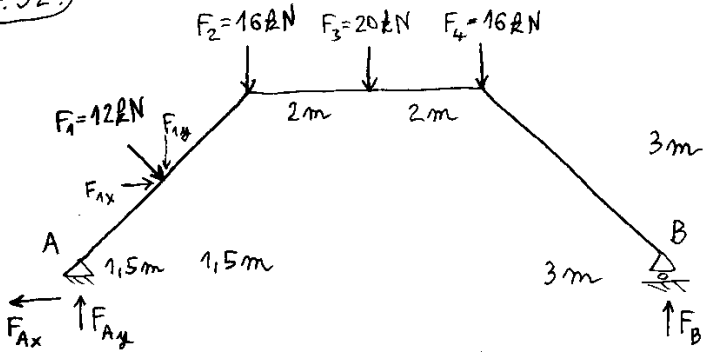
$\sum F_y = 0 = F_{Ay} - F_1 - F_2 - F_3 + F_B$

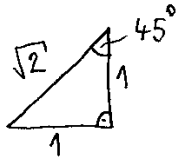
$$\boxed{F_{Ay} = F_1 + F_2 + F_3 - F_B = 2 + 6 + 6 - 5{,}833 = 8{,}167 \text{ kN } (\uparrow)}$$



F.4.32. Számítsuk ki a reakcióerőket egymásra halmozással, felbontva a terheket szimmetrikus és nem szimmetrikus két csoportra!

F.4.32.





$\sum F_x = 0 = -F_{Ax} + F_{1x} \rightarrow \boxed{F_{Ax} = F_{1x} = F_1 \frac{1}{\sqrt{2}} = 12 \frac{1}{\sqrt{2}} = 8{,}485\ \text{kN}\ (\leftarrow)}$

$\sum M_A = 0 = -F_1 \cdot 1{,}5\sqrt{2} - F_2 \cdot 3 - F_3 \cdot 5 - F_4 \cdot 7 + F_B \cdot 10$

$\boxed{F_B = \frac{1{,}5\sqrt{2}F_1 + 3F_2 + 5F_3 + 7F_4}{10} = \frac{1{,}5\sqrt{2} \cdot 12 + 3 \cdot 16 + 5 \cdot 20 + 7 \cdot 16}{10} = 28{,}55\ \text{kN}\ (\uparrow)}$

$\sum F_y = 0 = F_{Ay} - F_{1y} - F_2 - F_3 - F_4 + F_B$

$\boxed{F_{Ay} = F_{1y} + F_2 + F_3 + F_4 - F_B = 12 \frac{1}{\sqrt{2}} + 16 + 20 + 16 - 28{,}55 = 31{,}94\ \text{kN}\ (\uparrow)}$