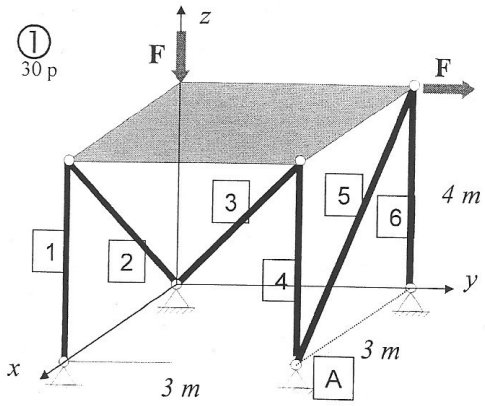
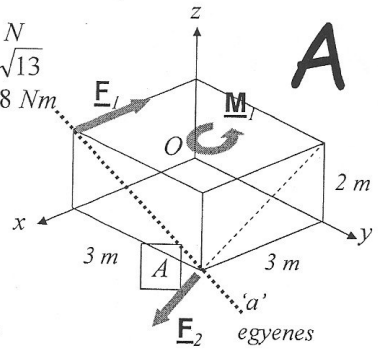
2009/10 őszi félév 1. zh.

① 30 p



Mekkora az A támaszerő?

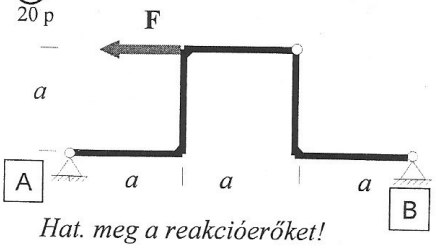
② 30 p $F_1 = 6\ N$ $F_2 = 2\sqrt{13}$ $M_1 = 18\ Nm$



$M_a = ?$

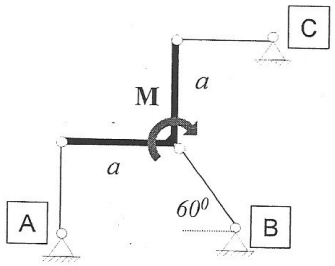
Hat. meg az erőrendszer legegyszerűbb alakját!

③ 20 p



Hat. meg a reakcióerőket!
adott: F, a

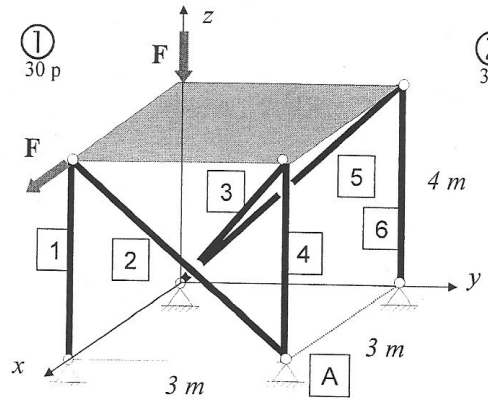
④ 20 p



Hat. meg a reakcióerőket!
adott: M, a

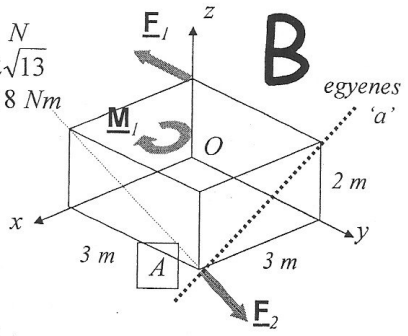
2009. okt. 13.

① 30 p



Mekkora az A támaszerő?

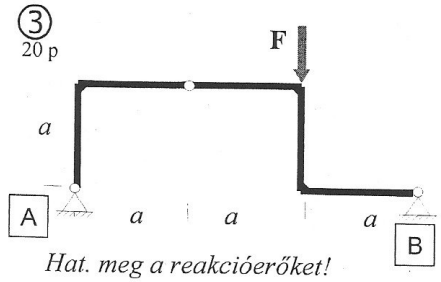
② 30 p $F_1 = 6\ N$ $F_2 = 2\sqrt{13}$ $M_1 = 18\ Nm$



$M_a = ?$

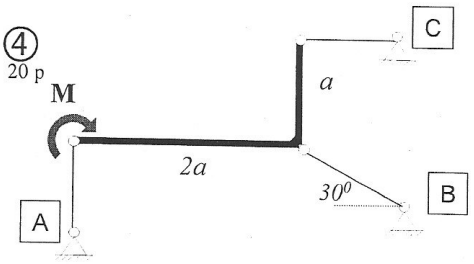
Hat. meg az erőrendszer legegyszerűbb alakját!

③ 20 p



Hat. meg a reakcióerőket!
adott: F, a

④ 20 p

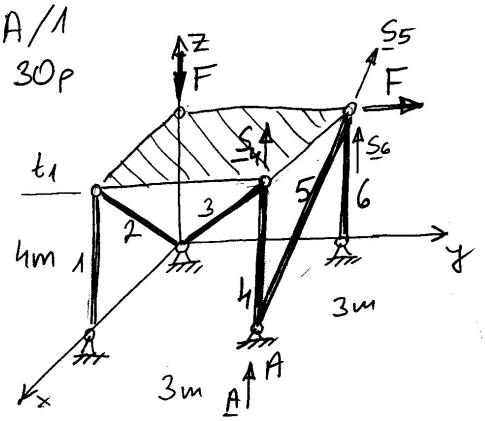


Hat. meg a reakcióerőket!
adott: M, a

2009. okt. 13.

2009/10 őszi félév 1. zh.

A/1
30p



$\underline{A} = ?$

$\sum M_z = 0 \quad S_{5x} \cdot 3 = 0$

$S_{5x} = 0$

$S_5 = 0\,N$

$\sum M_{t1} = 0 \quad -F \cdot 3 + S_6 \cdot 3 = 0$

$S_6 = F$ nyomott

$\sum M_x = 0 \quad -F \cdot 4 + S_4 \cdot 3 + S_6 \cdot 3 = 0$

$S_4 = \frac{4F - 3F}{3} = \frac{1}{3}F$ nyomott

$\underline{A} = \underline{S}_4 + \underline{S}_5 = \frac{1}{3}F \cdot \underline{k} + \underline{0} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{3}F \end{bmatrix} N$

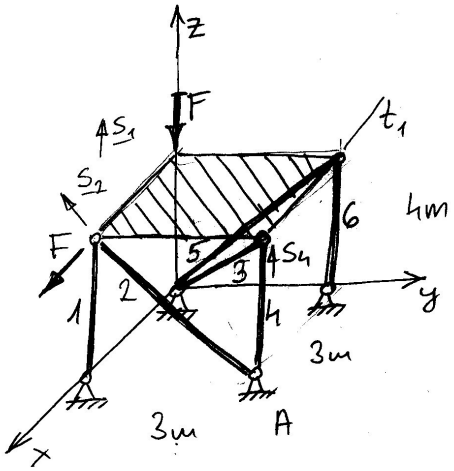
$|\underline{A}| = \frac{1}{3}F$

($S_{3y} = F$ húzott : $S_3 = F \cdot \frac{\sqrt{34}}{3} = 1{,}944F$

$S_1 = \frac{1}{3}F$ húzott

$S_{2x} = F$ nyomott : $S_2 = F \cdot \frac{5}{3}$)

B/1
30p



$\underline{A} = ?$

$\sum M_z = 0 \quad -S_{2y} \cdot 3 = 0$

$S_{2y} = 0 \quad S_2 = 0\,N$

$\sum M_{t1} = 0 \quad F \cdot 3 - S_1 \cdot 3 = 0$

$S_1 = F$ nyomott

$\sum M_y = 0 \quad F \cdot 4 - S_1 \cdot 3 - S_4 \cdot 3 = 0$

$S_4 = \frac{F}{3}$ nyomott

$\underline{A} = \underline{S}_2 + \underline{S}_4 = \begin{bmatrix} 0 \\ 0 \\ \frac{F}{3} \end{bmatrix}$

$|\underline{A}| = \frac{F}{3}$

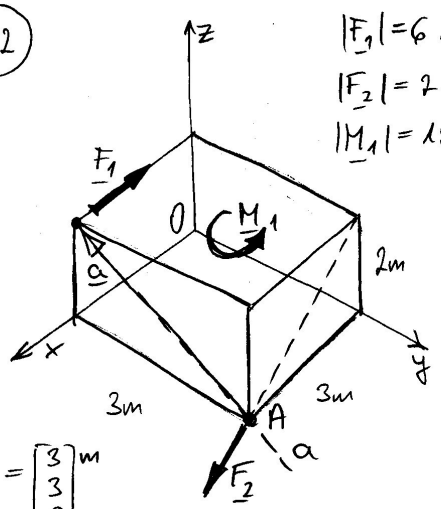
($S_6 = \frac{F}{3}\,N$ húzott

$S_{3x} = F$ húzott : $S_3 = \frac{\sqrt{34}}{3} \cdot F = 1{,}944F$

$S_{5y} = S_{3y} = F$ nyomott : $S_5 = \frac{5}{3}F$)

2009/10 őszi félév 1. zh.

A 2



$|\underline{F}_1| = 6\ N$ $\quad |\underline{F}_2| = 2\sqrt{13}$ $\quad |\underline{M}_1| = 18\ Nm$ $\quad M_a = ?$ $\quad$ legközelebb alak?

$\underline{F}_A = \underline{F}_1 + \underline{F}_2 = \begin{bmatrix} 0 \\ 0 \\ -4 \end{bmatrix} N$

$\underline{M}_A = \sum \underline{M}_j + \sum \underline{r}_{Ai} \times \underline{F}_i = \begin{bmatrix} 0 \\ 0 \\ 18 \end{bmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & -3 & 2 \\ -6 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & 0 \\ 6 & 0 & -4 \end{vmatrix} = \begin{bmatrix} 0 \\ 0 \\ 18 \end{bmatrix} + \begin{bmatrix} 0 \\ -12 \\ -18 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -12 \\ 0 \end{bmatrix} Nm$

$\underline{r}_{OA} = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} m$

$M_a = \underline{M}_A \cdot \underline{e}_a = \dfrac{0 \cdot 0 + -12 \cdot -3 + 0 \cdot 2}{\sqrt{13}} = \dfrac{36}{\sqrt{13}} = 9{,}985\ Nm$

$\underline{F}_1 = \begin{bmatrix} -6 \\ 0 \\ 0 \end{bmatrix} N$ $\quad \underline{F}_2 = \begin{bmatrix} 6 \\ 0 \\ -4 \end{bmatrix} N$

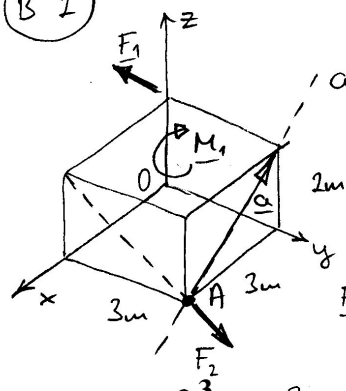
$\underline{r}_{AC} = \dfrac{\underline{F}_A \times \underline{M}_A}{|\underline{F}_A|^2} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & -4 \\ 0 & -12 & 0 \end{vmatrix} \cdot \dfrac{1}{4^2} = \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix} m$

$\dfrac{F_{2x}}{F_2} = \dfrac{3}{\sqrt{13}}$ $\quad \underline{e}_a = \dfrac{\underline{a}}{|\underline{a}|} = \dfrac{\begin{bmatrix} 0 \\ -3 \\ 2 \end{bmatrix}}{\sqrt{13}}$

$\underline{r}_{OC} = \underline{r}_{OA} + \underline{r}_{AC} = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} m$

$\dfrac{F_{2z}}{F_2} = \dfrac{2}{\sqrt{13}}$ $\quad \underline{F}_C = \underline{F}_A$ $\quad \underline{M}_f = \dfrac{\underline{F}_A \cdot \underline{M}_A}{|\underline{F}_A|^2} \cdot \underline{F}_A = \dfrac{0 \cdot 0 + 0 \cdot -12 + -4 \cdot 0}{4^2} \cdot \begin{bmatrix} 0 \\ 0 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} Nm$

B 2



$|\underline{F}_1| = 6N$ $\quad |\underline{F}_1| = 2\sqrt{13}\ N$ $\quad |\underline{M}_1| = 18Nm$ $\quad M_a = ?$ $\quad$ legközelebb alak?

$\underline{F}_1 = \begin{bmatrix} 0 \\ -6 \\ 0 \end{bmatrix} N$ $\quad \underline{F}_2 = \begin{bmatrix} 0 \\ 6 \\ -4 \end{bmatrix} N$

$\dfrac{F_{2y}}{F_2} = \dfrac{3}{\sqrt{13}}$; $\dfrac{F_{2z}}{F_2} = \dfrac{2}{\sqrt{13}}$

$\underline{r}_{OA} = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} m$

$\underline{M}_A = \sum \underline{M}_j + \sum \underline{r}_{Ai} \times \underline{F}_i = \begin{bmatrix} 0 \\ 0 \\ -18 \end{bmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -3 & -3 & 2 \\ 0 & -6 & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & 0 \\ 0 & 6 & -4 \end{vmatrix} = \begin{bmatrix} 0 \\ 0 \\ -18 \end{bmatrix} + \begin{bmatrix} 12 \\ 0 \\ 18 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} Nm$

$\underline{F}_A = \underline{F}_1 + \underline{F}_2 = \begin{bmatrix} 0 \\ 0 \\ -4 \end{bmatrix} N$

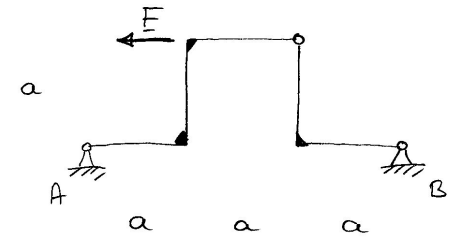
$\underline{e}_a = \dfrac{\underline{a}}{|\underline{a}|} = \dfrac{\begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}}{\sqrt{3^2 + 2^2}}$ $\quad M_a = \underline{M}_A \cdot \underline{e}_a = \dfrac{12 \cdot -3 + 0 \cdot 0 + 0 \cdot 2}{\sqrt{13}} = \dfrac{-36}{\sqrt{13}} = -9{,}985 Nm$

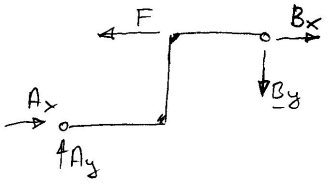
$\underline{r}_{AC} = \dfrac{\underline{F}_A \times \underline{M}_A}{|\underline{F}_A|^2} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & -4 \\ 12 & 0 & 0 \end{vmatrix} \cdot \dfrac{1}{4^2} = \begin{bmatrix} 0 \\ -3 \\ 0 \end{bmatrix} m$ $\quad \underline{r}_{OC} = \underline{r}_{OA} + \underline{r}_{AC} = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} m$

$\underline{F}_C = \underline{F}_A$; $\quad \underline{M}_f = \dfrac{\underline{F}_A \cdot \underline{M}_A}{|\underline{F}_A|^2} \cdot \underline{F}_A = \dfrac{0 \cdot 12 + 0 \cdot 0 + -4 \cdot 0}{4^2} \begin{bmatrix} 0 \\ 0 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} Nm$

2009/10 őszi félév 1. zh.

A 3) Hat. meg. a reakcióerőket!
Adott: F ; a





$|\underline{B}_x| = |\underline{B}_y|$

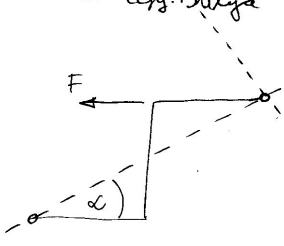
$\Sigma M_A = 0 \quad F \cdot a - B_y \cdot 2a - B_x \cdot a = 0$

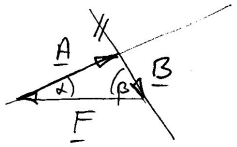
$F \cdot a - 3a \cdot B_x = 0$

$B_x = B_y = \frac{F}{3}$

$|\underline{B}| = \frac{\sqrt{2}}{3} F$

3 erő egyensúlya





2 erő egyensúlya

$\operatorname{tg}\alpha = \frac{a}{2a} = \frac{1}{2}$

$\alpha = 26{,}565°$

$\operatorname{tg}\beta = \frac{a}{a} = 1$

$\beta = 45°$

$\Sigma F_x = 0 \quad A_x - F + B_x = 0$

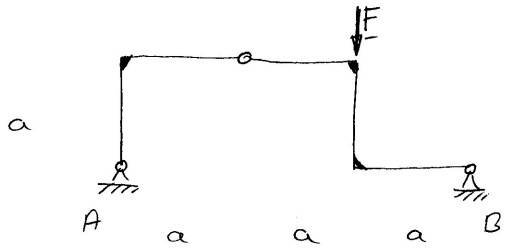
$A_x = \frac{2}{3} F \rightarrow$

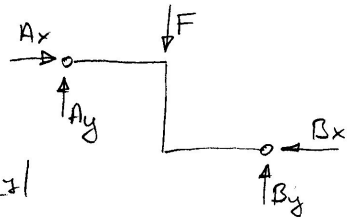
$\Sigma F_y = 0 \quad A_y - B_y = 0$

$A_y = B_y = \frac{F}{3} \uparrow$

$|\underline{A}| = \frac{\sqrt{10}}{3} F$

B 3) Hat. meg. a reakcióerőket!





$|\underline{A}_x| = |\underline{A}_y|$

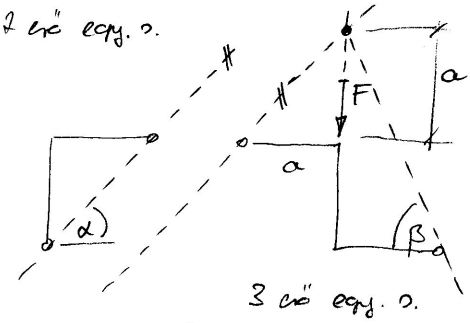
$\Sigma M_B = 0 \quad -A_x \cdot a - A_y \cdot 2a + F \cdot a = 0$

$-3A_x \cdot a + F \cdot a = 0$

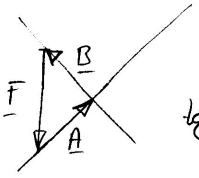
$A_x = A_y = \frac{F}{3}$

$|\underline{A}| = \frac{\sqrt{2}}{3} \cdot F$

2 erő egyensúlya



3 erő egyensúlya



$\alpha = 45°$

$\operatorname{tg}\beta = \frac{2a}{a} = 2$

$\beta = 63{,}435°$

$\Sigma F_x = 0 \quad A_x - B_x = 0$

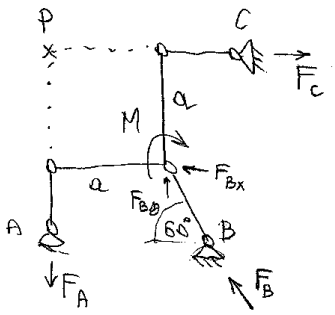
$B_x = A_x = \frac{F}{3} \leftarrow$

$\Sigma F_y = 0 \quad A_y - F + B_y = 0$

$B_y = \frac{2}{3} F \uparrow \quad |\underline{B}| = \frac{\sqrt{10}}{3} F$

2009/10 őszi félév 1. zh.

A4



$$F_{Bx} = F_B \cos 60^\circ = \frac{1}{2} F_B$$

$$F_{By} = F \sin 60^\circ = \frac{\sqrt{3}}{2} F_B$$

$$\sum M_P = 0 = -M - F_{Bx}\, a + F_{By}\, a =$$

$$= -M - \frac{1}{2} F_B\, a + \frac{\sqrt{3}}{2} F_B\, a$$

$$\boxed{F_B = \frac{2}{\sqrt{3}-1} \frac{M}{a} = 2{,}732 \frac{M}{a} \ (\uparrow)}$$

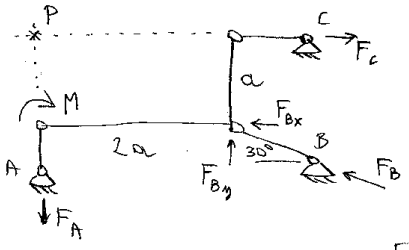
$$F_{Bx} = \frac{1}{2} \frac{2}{\sqrt{3}-1} \frac{M}{a} = \frac{1}{\sqrt{3}-1} \frac{M}{a} = 1{,}366 \frac{M}{a}$$

$$F_{By} = \frac{\sqrt{3}}{2} \frac{2}{\sqrt{3}-1} \frac{M}{a} = \frac{3}{3-\sqrt{3}} \frac{M}{a} = 2{,}366 \frac{M}{a}$$

$$\sum F_x = 0 = F_C - F_{Bx} \rightarrow \boxed{F_C = F_{Bx} = 2{,}366 \frac{M}{a} \ (\rightarrow)}$$

$$\sum F_y = 0 = -F_A + F_{By} \rightarrow \boxed{F_A = F_{By} = 1{,}366 \frac{M}{a} \ (\downarrow)}$$

B4



$$F_{Bx} = F_B \cos 30^\circ = \frac{\sqrt{3}}{2} F_B$$

$$F_{By} = F_B \sin 30^\circ = \frac{1}{2} F_B$$

$$\sum M_P = 0 = -M - F_{Bx}\, a + F_{By} \cdot 2a =$$

$$= -M - \frac{\sqrt{3}}{2} F_B\, a + \frac{1}{2} F_B \cdot 2a$$

$$\boxed{F_B = \frac{2}{2-\sqrt{3}} \frac{M}{a} = 7{,}464 \frac{M}{a} \ (\nwarrow)}$$

$$F_{Bx} = \frac{\sqrt{3}}{2} \frac{2}{2-\sqrt{3}} \frac{M}{a} = \frac{\sqrt{3}}{2-\sqrt{3}} \frac{M}{a} = 6{,}464 \frac{M}{a}$$

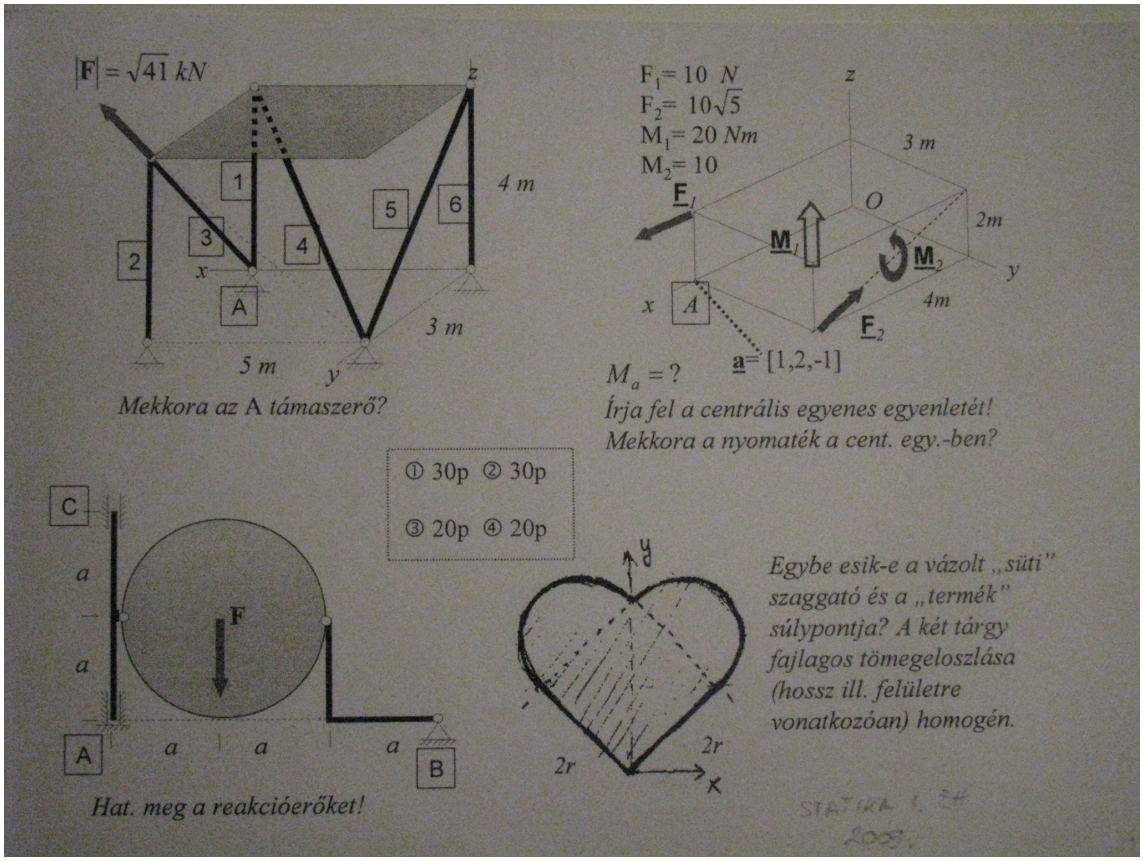
$$F_{By} = \frac{1}{2} \frac{2}{2-\sqrt{3}} \frac{M}{a} = \frac{1}{2-\sqrt{3}} \frac{M}{a} = 3{,}732 \frac{M}{a}$$

$$\sum F_x = 0 = F_C - F_{Bx} \rightarrow \boxed{F_C = F_{Bx} = 6{,}464 \frac{M}{a} \ (\rightarrow)}$$

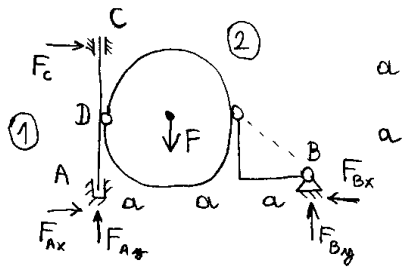
$$\sum F_y = 0 = -F_A + F_{By} \rightarrow \boxed{F_A = F_{By} = 3{,}732 \frac{M}{a} \ (\downarrow)}$$

Egy régebbi zh feladatsor:

A súlypontos feladat most nincs benne az 1. zh anyagában.



A 3. feladat megoldása számítással, erőkomponensekkel:



1.) $\sum M_D^{(2)} = 0 = -Fa + F_{By} \cdot 3a - F_{Bx}\, a$

2.) $F_{Bx} = F_{By}$

2.→1.) $0 = -Fa + F_{By} \cdot 3a - F_{By}\, a$

$$\boxed{F_{By} = \frac{1}{2} F\ (\uparrow)}$$

2.) $$\boxed{F_{Bx} = F_{By} = \frac{1}{2} F\ (\leftarrow)}$$

$\sum F_y = 0 = F_{Ay} - F + F_{By}$

$$\boxed{F_{Ay} = F - F_{By} = F - \frac{1}{2} F = \frac{1}{2} F\ (\uparrow)}$$

$\sum M_A = 0 = -F_C \cdot 2a - Fa + F_{By} \cdot 3a$

$$\boxed{F_C = \frac{-F + 3F_{By}}{2} = \frac{-F + 3\frac{1}{2}F}{2} = \frac{1}{4} F\ (\rightarrow)}$$

$\sum F_x = 0 = F_{Ax} + F_C - F_{Bx}$

$$\boxed{F_{Ax} = F_{Bx} - F_C = \frac{1}{2} F - \frac{1}{4} F = \frac{1}{4} F\ (\rightarrow)}$$